

Analysis of a Turkish Mathematics Textbook in the Late Ottoman Era with respect to the Principles of Contemporary Elementary Education

Osmanlı Geç Dönemine ait bir Türkçe Matematik Ders Kitabının Çağdaş İlköğretim Eğitiminin Dayanakları Açısından İncelenmesi

Esra YAPRAK*

M. Sencer ÇORLU**

Emin AYDIN***

Abstract. Developing an understanding of the foundations of the educational tradition of the Turkish Republic is connected to an exploration of the specifics of Ottoman education. This qualitative study explored an Ottoman mathematics textbook published in the early twentieth century. Under the influence of naturalistic inquiry, the textbook was analyzed in terms of content, organization, and principles of elementary mathematics education. It was concluded that the textbook is successfully presented multiple representations and real-life examples while the development of content did not provide opportunities to develop reasoning skills.

Keywords: Mathematics Education, History of Education, Textbook Analysis.

Öz. Türkiye Cumhuriyeti'nin eğitim geleneğinin temellerinin anlaşılması, Osmanlı Devletindeki eğitim anlayışının özelliklerinin araştırılmasına bağlıdır. Bu nitel çalışmada, yirminci yüzyılın başlarında yayınlanan bir Osmanlı matematik ders kitabı inceleme konusu edildi. Natüralist araştırma metodolojisinin etkisi altında, bu ders kitabı ilköğretim matematik eğitiminin içeriği, organizasyonu ve ilkeleri açısından analiz edilmiştir. Araştırmanın sonunda, ders kitabında çoklu temsiller ve gerçek yaşam örnekleri başarılı bir şekilde sunulurken, içeriğin akışı, akıl yürütme becerilerini geliştirmek için yeterli fırsatlar sağlamadığı sonucuna varılmıştır.

Anahtar Kelimeler: Matematik Eğitimi, Eğitim Tarihi, Ders Kitabı İncelemesi.

Public Interest Statement.

Understanding of the foundations of the educational tradition of the Turkish Republic can shed light to the current educational approaches. We aim to analyze a late Ottoman textbook with respect to the contemporary educational approaches. The findings indicated the textbook used approaches such as multiple representations and real life contexts which are in line with the current scientific evidence regarding mathematical learning.

Toplumsal Mesaj.

Türkiye Cumhuriyetinin eğitim geleneğinin anlaşılması bugünkü eğitim anlayışlarına ışık tutabilir. Bu amaçla Osmanlı devletinin son yıllarında yazılmış bir ders kitabının günümüz eğitim yaklaşımları açısından incelenmesi amaçlanmıştır. İnceleme sonucunda, ders kitabında öğrenmeyle ilgili bilimsel verilere uygun düşen çoklu temsil ve gerçek hayat örnekleri kullanımı gibi birçok yöntemin kullanılmış olduğu gözlenmiştir.

* Orcid ID: <https://orcid.org/0000-0001-7710-7242>, İhsan Doğramacı Bilkent University, Graduate School of Education. esra.yaprak@mfa.gov.tr

** Orcid ID: <https://orcid.org/0000-0002-1741-028X>, Bahçeşehir University, BAUSTEM. mehmetsencer.corlu@rc.bau.edu.tr

*** Orcid ID: <https://orcid.org/0000-0003-4298-2623>, Marmara University, Atatürk Faculty of Education. eyaydin@marmara.edu.tr

1. INTRODUCTION

There were several specific uses of mathematics in the daily lives of Ottomans: for example, inheritance problems (a branch of law), finding the direction for prayer, times of prayer, annual calculations of Eid times, time calculations, astronomy, and so on. The Ottomans also used mathematics in the encryption of tax calculations (siyakat) and in other encryption system, such as the abjad (ebced) alphabet. This indicates that everyday life as an Ottoman to an extent depended on mathematics. Due to the importance of mathematics in Ottoman daily life, it was a core subject at the elementary school level (maktab). Students at this level gained basic numeracy skills, which prepared them for secondary school level mathematics (İzgü, 1997).

1.1 Background

The Ottoman education system consisted of institutions that were inherited by the Seljuq Turks up until the 18th century. Elementary schools (maktab) and secondary-higher education schools (madrasa) provided education to young people in the Empire. There were also palace schools (Enderun Maktab) (Corlu, Burlbaw, Capraro, Corlu, & Han, 2010).

An elementary school (maktab) was based on a mosque-school system founded and supported by elite statesmen or sultans. Young learners began their education at those institutions with a ceremony called, literally translated, beautiful start (bed'besmele; İhsanoglu, 2002). All children had the right to attend school. Those who were educated in secondary-higher education institutions (madrasa), along with certain literate mosque caretakers, were selected as elementary school teachers. Maktab had mainly religious purposes. They taught reading and writing of the alphabet, handwriting (calligraphy), the basic principles of Islam and the Quran, basic counting, and the four basic arithmetical operations, known as black sentence (İhsanoglu, 2002). The basic principles of these institutions were based on the ideas of sharing and helping other people, being respectful to others' ideas and opinions, being more tolerant of others, and behaving rationally as educated people. All those principles were intended to encourage young learners to become well-educated citizens (Sönmez, 2013). The maktab was essential for students who wished to continue their educations at the madrasa level. The madrasa, which mainly refers to secondary-higher education, included both religious and secular subjects (İhsanoglu, Chatzis & Nicolaidis, 2003). There were also palace schools (enderun maktab), which provided education for youngsters who were to become members of the administrative elite of the Ottoman society (Taşkın, 2008).

Towards the end of the 18th century, the performance of the maktab, madrasas, and enderun began to fall, as a result of changes due to the influence of intellectual and cultural ideas in Western Europe during the 17th century (Akyüz, 1993). More emphasis was being given to reason, logic, and analysis in the West. There was also much talk about science, toleration, and skepticism. Those ideas spread throughout the continent. As a result, the Ottoman education was affected by those Western ideologies (Lewis, 2001).

Most traditional educational methods have been disputed during this era and modern educational philosophies have been developed, which are in contrast to traditional approaches. The importance of scientific knowledge and intellectual expression has increased (Mardin, 1960). Modern educational philosophies spread all around the world, including throughout the Ottoman Empire, in the 18th century (Lewis, 2001). Modern educational ideas began to influence the worldview of Ottoman elites, causing concerns about the faith of the Empire. They identified country's main deficiency to lie in war technology and pressured Ottoman Sultans to reform the army. Changes then began, starting with military (Göçek, 1996; Lewis, 1968). A long period of reform also revealed deficiencies in qualified human resources for implementing reforms. This led the Ottoman educators to focus on educating new generations on the basis of contemporary education philosophies and principles. They believed that conventional education methods required changes, because traditional methods had not been satisfactory (Somel, 2001).

The Tanzimat period (1839-1856) was characterized by attempts to establish graded systems of schooling, which were different in many ways from traditional institutions (Kazamias, 1969). Traditional institutions were far from effective and sufficient for educating young people (Şanal, 2003). Many stakeholders (such as government, teachers, and parents) started to be aware of a need to increase the quality of teaching and learning, particularly in science, engineering, medicine, and mathematics (Cemaloğlu, 2005). Instead of abolishing ineffective institutions, policymakers decided to introduce Western style institutions alongside the traditional institutions, thus creating a dual system.

The dual system was initiated in 1869 by Mehmet Esad Safvet Pasha (1814-1883), then the Minister of Education. It was called the General Education Act (Maarif-i Umumiye Nizamnamesi; Somel, 2001). The Ministry of Education started to open new institutions for training youngsters. The dual education system was divided into three parts: primary school education (sibyan schools and rushdiyes), secondary school education (idadis and sultanis), and higher education (Darulfunun; Göçek, 1996; Kazamias, 1969). New regulations gradually spread to the whole state; these became the foundation upon which later reforms were introduced during the early Republican period, that is 1923-1938 period (Aslan & Olkun, 2011).

1.2 Problem

Despite increased interest in Ottoman life and language in recent years in Turkey, little research has been conducted on their educational systems. In particular, very few studies have focused on how textbooks were prepared to interact strategically with teachers and students in mathematics education (Aydin, et al., 2017). Only a couple of analyses have focused on the Ottoman mathematics textbooks (Yel, 2010; Ceylan & Özdemir, 2012; Özdemir & Ceylan, 2017; Yilmaz & Ozyigit, 2017). Thus, there is a need to understand the foundations of the educational tradition of the Turkish Republic (and perhaps other independent states) by exploring the specifics of Ottoman education. Mathematics education is one of the subjects that can reveal the development of educational traditions. Given the fact that written documents can serve as witnesses of historical periods, it is critical to analyze a historical textbook that contains an incredible amount of facts, data, and cultural information. As Schissler (1990) indicated, "Textbooks convey a global understanding of history and of the rules of society as well as norms of living with other people" (p. 81). We feel that there is a need for analyses of Ottoman textbooks published in the 19th and 20th centuries. The purpose of this study, therefore, was to explore a mathematics textbook published during the modernization period of the Ottoman Empire (1828-1908). This study analyzes the textbook in terms of its content and organization, as well as its instructional methods with no comparison to any other state or textbook.

1.3 Research Questions

In a qualitative study of exploratory nature, the research questions formulated at the beginning of the study can change with the emerging themes from the data analysis. Therefore first iterations of questions are tentative. This gives a flexible starting point for the researchers (Agee, 2009). In order to achieve the purpose of the study, the researchers started with a set of four tentative research questions about teaching, learning, and assessment in an elementary school mathematics textbook:

- How does the textbook content take into account the developmental levels of students?
- How do the problems and exercises throughout the textbook address student's developmental level?
- What evidence is there for multiple representations of mathematical structures?

Does the author approach mathematics holistically, with a focus on investigations and reasoning, or in a more procedural fashion?

After the content analysis, with the emergence of four themes which will be discussed in the following sections, the researchers re-formulated the research questions as follows:

- Research Question 1: What skills and concepts does the book is intened to develop?
- Research Question 2: What is the relative weight given to the procedural or conceptual understanding?
- Research Question 3: How are the levels of difficulty of the questions were managed throughout the textbook?

2. METHOD

A naturalistic paradigm of inquiry was used to carry out this study. Cohen, Manion, and Morrison (2007) stated that the selection of the design for a study should be led by identifying the problem and research purposes. Once the focus was shaped, the theoretical framework emerged from the inquiry and the methodology was designed. The naturalistic inquiry was the most appropriate strategy (Lincoln & Guba, 1985).

The study uses a historical perspective. A historical analysis is generally discussed in terms of authenticity, meaning, and theorization (McCulloch, 2005). Firstly, the authenticity is provided by alerting some of the inconsistencies within the textbook itself. The researchers are aware of there is no possibility of an informed judgment about the data. Secondly, different kind of pieces of evidence for the data investigated from historical source. The information in the textbook compared and considered together with other research. Thirdly, the textbook has its own language in terms of mathematical terms and particular Ottoman-Turkish words. The usage of these concepts have understood as its contemporaries would be understood it, rather than as it would be understood today. Lastly, the interpretive outlook stressed while analyzing the textbook. The interpretation of data established in terms of its symbolic structures and contextual determination of meaning

2.1 Sampling

The purposeful sample was an Ottoman mathematics textbook that was published in the early 20th century (Lincoln & Guba, 1985). The sample textbook was selected based on several criteria: accessibility to the researcher, published date of the textbook, and number of pages due to time constraints (Cohen, Manion, & Morrison, 2007). Moreover, sample comes with supporting historical documents (Ottoman-Turkish/modern Turkish mathematics dictionary). The sample was the third edition of a historical mathematics textbook to be used in elementary school (Iptidai School) in 1326 (1908 for Hijri calendar or 1910/1911 for Rumi calendar). It was published by Artin Asaduryan Mahdumlari (Artin Asaduryan & His Sons) Matbaasi (Artin Asaduryan & His Sons) in Istanbul, Turkey. The textbook was purchased from an online bookstore through auction. The textbook consisted of 81 pages. Because of circumstances beyond control during data collection (time constraints, limited the financial resources for translation), the current study includes only one textbook.

2.2 Data Collection

Several different methods were employed to collect the data. Due to logistic reasons, including accessibility and availability of the document, data were collected via an online bookstore. The book was, then, translated from Ottoman Turkish to the modern Turkish language by a mathematics graduate who is also knowledgeable in the Ottoman-Turkish language. A reflective journal was kept throughout the process in order to increase the reliability of the interpretations. The journal included notes on the translator, the history expert, and a peer debriefer, as well as notes on literature findings that we used to construct a working hypothesis. To ensure that reliability of the data several techniques were incorporated into the study: We used multiple historical sources to increase the probability of producing credible findings, and the information in these sources provided external checks on the inquiry process. An expert in Ottoman historical textbooks was also consulted during interpretation of the data.

2.3 Data Analysis

The translated history textbook was subjected to qualitative analysis (Gall, Borg, & Gall, 2003) by using content analysis (Creswell, 2011). The content analysis steps followed in the current study based on Creswell's framework are: (a) unitizing data; (b) labeling the unitized data with codes; (c) categorization of the codes; (d) identifying themes (Creswell, 2011, p. 244).

Eventually, various categories were created by utilizing data (Lincoln & Guba, 1985). At the end, each category was reviewed and possible categories were reconsidered for accuracy. The final categories were: (1) the scope of the textbook; (2) procedural knowledge; (3) real life examples; (4) number concept; (5) multiple representations (6) problems with basic operations; (7) reasoning strategies; (8) measurement units; (9) addition facts; (10) multiplication facts; (11) subtraction facts; (12) division facts; (13) exercises with basic operations; (14) challenging tasks; (15) drill exercises. The four themes that emerged were (I) number sense skills, (II) procedural mastery in basic arithmetic operations, (III) the concept of measurement and procedural skills and (IV) level of challenge in the questions.

3. RESULTS

This section includes the results of the study collected from the data and rich information connected to the findings of the data analysis in regard to the research questions on the basis of the four themes that were identified: number sense skills, procedural mastery in basic arithmetic operations, the concept of measurement, and problem solving skills. The findings from the theme 1 and theme 2 explore the first research question. Second and third research questions are explored using the findings from the theme 3 and theme 4 respectively.

3.1 Theme I: Number sense skills

The first example involves the use of strategies to develop students' learning of the concepts of counting. It was found that the concepts of less and more were presented while introducing numbers (Broody, 1987). Van de Walle, Karp, & Bay-Williams (2010) emphasized the difficulty of the less concept compared to the more, and analysis indicated that textbook allows students to get more exposure to the word more than less. A second example concerns the use of numbers as anchors or benchmarks. The author encouraged the use of 10 as a benchmark in order to develop the relationship among numbers. The textbook categorized numbers in groups of 10 (Figure 1).

٣	٢	١	3	2	1
اوج	ایکی	بر	Three	Two	One
٦	٥	٤	6	5	4
القی	پن	دوت	Six	Five	Four
٩	٨	٧	9	8	7
طورو	سکڑ	پدی	Nine	Eight	Seven

Figure 1. Numbers 1 to 10.

A third example concerns the use of real life examples or pictures. Van de Walle et al. (2010) suggested that real life examples are useful to indicate numbers, including zero, which was followed by its symbolic representation. While numbers up to 10 were first symbolized with bird pictures—the zero concept was indicated with a no bird image. However, this characteristic of zero may mean for the student a lack of a characteristic and that may lead to a misinterpretation of all

numbers that includes the zero symbol. For example, 10 (ten) is likely to be interpreted as 1 (Anghileri, 2006).

The textbook uses the cardinality principle. The way that this principle is used in the textbook allows the student to match counting words with objects one by one. This principle; however, is only used in smaller numbers. When the numbers got bigger, for example for 20 to 30, it is assumed that the student had already understood that the last number in a set has a special meaning; the last number represents the number of elements of the set. This is compatible with the developmental levels of the students (Nikoloska, 2009; Van de Walle et al., 2010). The Figure 2 represented number ten as the whole in the pears set:

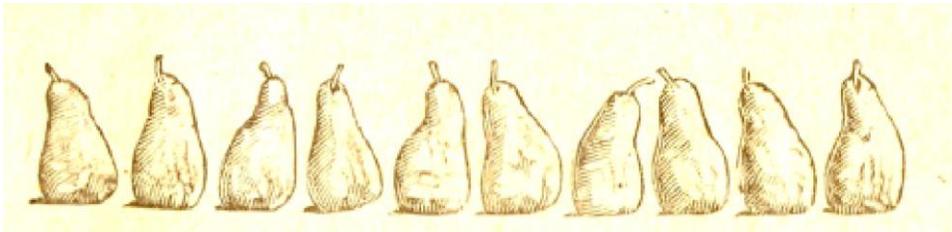


Figure 2. Ten (10) pears.

The textbook mentions the base-ten system in a developmental way; decimal places are presented while numbers are introduced in an increasing order. However, there is no comprehensive explanation about decimal places throughout the textbook. The textbook assumes that students can develop an overall understanding of decimal places from particular examples. These particular examples includes decimal places up to thousands which are introduced by using their location on a number—the ones on the right and the tens and hundreds places to the left. By doing so, the textbook shows the difference between consecutive decimal places.

As illustrated in Figure 3, the textbook includes multiple representations for introducing numbers; this example was chosen from place values topic. Splitting the numbers into tens and ones, is expressed using images and written language as well as expressing eleven as comprised of 1 ten and 1 one. Van de Walle et al. (2010) stated that the more ways that are given to think about a concept, the more learners integrate the concepts in a meaningful manner. Thus, this repetition can be useful that might promote students' awareness and critical thinking abilities (Aiken, 1972; Carpenter & Lehrer, 1999).

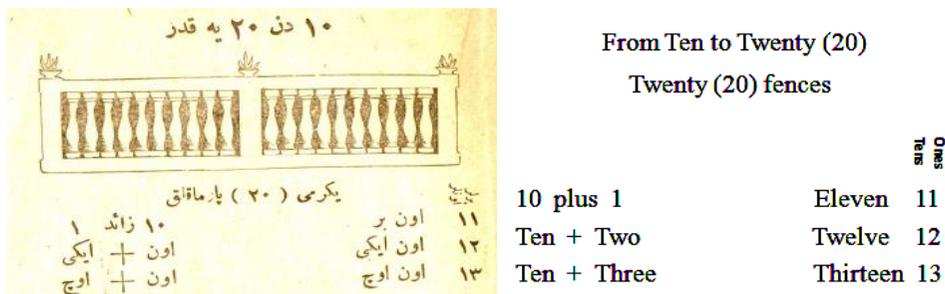


Figure 3. Units in numbers.

3.2 Theme II. Procedural mastery in basic arithmetic operations

Data analyses revealed that the textbook combines the introduction of basic arithmetic operations with numbers. Numbers are constructed (1-1000) by indicating their relationships in terms of arithmetic operations. Particularly, addition and multiplication are used more than other basic operations.

3.2.1 Findings related to the addition concept

The addition concept is combined with numbers by consecutive counting. Interestingly, in the textbook, addition is not explicitly defined first; instead, only some exercises and consecutive counting are initially employed, which harks back to Van de Walle et al. (2010), who stated that exemplifying concepts instead of providing direct definitions is effective to illuminate the logic behind the concepts. Thus, this strategy stimulates youngsters to explore the concepts independently.

Formalizing language is frequently used in the addition exercises. In the textbook, addition is represented as one more than, two more than, make more, plus, and add. It was interesting that the textbook introduced the mathematical notation, precisely the plus word and its (+) symbol in the second example, which followed an example with only written language. These two examples show that the verbal and symbolic representation are used in a developmental way. Symbolization help young children to remember more easily, as proposed by Carpenter and Moser (1984). The language used for the operations is instrumental in facilitating the early stages of conceptual understanding (Anghileri, 2006). The following exercises from the textbook exemplify this concept:

1) *If there is one finger and one finger more, how many fingers do you have? (p. 9)*

2) *If there are two fingers and two fingers more, how many fingers do you have? (p. 9)*

The addition concept is properly defined only through the end of the textbook. It is illustrated in the textbook using the following example:

I am adding 3 walnuts to 6 walnuts. That means that I am doing addition. When I added those altogether, I had 9 walnuts. This is called the total (yekun). The symbol of addition is +, called plus or more (p. 11).

Once the one more strategy is employed, the two more than strategy is linked with even and odd numbers for two digit numbers. The concept of adding two is treated by researchers as a useful practical activity (Carpenter & Moser, 1984; Anghileri, 2006; Van de Walle et al., 2010), especially as larger numbers such as 30 and 2 more or 57 and 2 more establish proficiency in counting. Five is also used as an anchor; fingers in various counting and addition exercises demonstrate this while adding up to over 50.

It is surprising to observe that the structure of this textbook seems to follow an inductive approach while explaining the addition concept: Examples and problems constitute the first step of instruction in the textbook, followed by the rules and the exercises. That way "the student thinks through several examples and then generalizes a rule at the end" (Brahier, 2013; p.63). In addition, the question-answer teaching strategy is employed to specify certain elements of addition. The following is an example from the textbook in which the textbook seems to expect that the students deduct the addition concept by themselves with the addition elements and some examples:

What is addition? Addition is adding the same kinds of things together and reporting the result with the same kinds. How do you write the numbers that you will add together? Write the ones under the ones, the tens under the tens, and the hundreds under the hundreds (p. 39).

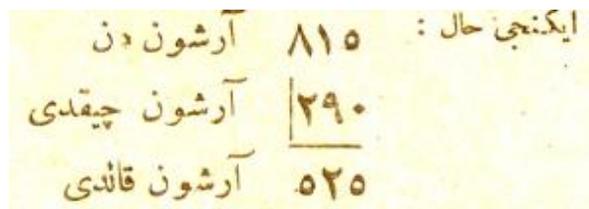
3.2.2 Findings related to the subtraction concept

In the textbook, subtraction is illustrated by employing the addition concept. The first reasoning strategy is the one less strategy that is used in number counting, which accompanies the one more strategy. The textbook compares these two strategies by using the same numbers; this approach is more effective because of the difficulties of subtraction, and children can grasp the concept more clearly (Thornton & Toohey, 1985). Although adding zero is not specifically explained in numbers, zero is employed in many subtraction exercises. As previously mentioned in the addition section, zero is associated with the no bird scenario, which establishes an understanding of the conceptual meaning of zero. In addition, the finger method (using fingers for arithmetic operations) is

implemented for subtraction by asking "How many fingers are left if you close 3 out of 5 fingers (p. 9)?".

In referring to the concept, instead of subtract, the word close was preferred. Formalizing phrases such as take away and decreasing are other language preferences used in the textbook. However, there is a danger that such uses might lead conceptual misunderstandings: 4 less than a number can be understood as 4 subtracted from any given number while at the same time it can be understood as any given number subtracted from 4 (Capraro, Capraro & Rupley, 2011). To explain the concept of subtraction the author had preferred to describe how the division algorithm was carried out and no other conceptual explanation seemed to have provided in which a question & answer strategy is used in a didactical manner (Figure 4):

Subtract zero from five; it is five; I write five under the line; 1 is less than 9; the difference between 11 and 9 is 2. I write 2 under the line and you have 1. You have 2, which with the addend 1, equals 3. The difference between 8 and 3 is 5(p.44).



$$\begin{array}{r} \text{Second Case: } 815 \text{ cubits} \\ | \quad 290 \text{ cubits take away} \\ \hline 525 \text{ cubits left} \end{array}$$

Figure 4. Subtraction

The textbook uses the two less strategy in the second stage. This helps students to understand the concept of subtraction while they use two as a subtrahend. This reasoning strategy might help in solving multi-digit problems (Van de Walle et al., 2010). The two less strategy is employed for introducing odd and even numbers, and children can compare addition and subtraction when they are inverse and reversible.

3.2.3 Findings related to the multiplication and division concepts

The relationship between addition and multiplication is emphasized; it seems, with the aim of showing a system of patterns. Multiplication is introduced first by using the doubles strategy (Van de Walle et al., 2010). In the following pages other numbers are used as factors: For example, 5 was used as a benchmark to illustrate the numbers from 1 to 50. Here it is possible to mention the existence of an effort to use a conceptual link between the earlier and the new concept. Zero is also represented as a factor. However, the zero effect in multiplication is not explained, and only the difference between addition and multiplication is demonstrated (e.g., $5 + 0$ stays the same, 5×0 is always zero).

The symbol and its oral expression as well as the elements of multiplication in number introduction are demonstrated, and multiplication and addition are comparatively explained using the same numbers. The symbols along with their oral expressions are also applied to these numbers (Angliheri, 2006). The term times is used as a formalizing term in the textbook, and the examples are as follows:

How many fingers are there if I have 3 times 3 fingers? How many fingers are there if I have 2 times 3 fingers (p. 77)?

The procedure of multiplication is divided into two parts. In the first part, a three digit number is multiplied by one digit. In the second part, three digits are multiplied by two digits. In addition, the multiplication algorithm is explained in a detailed stepwise manner as in the case of the division algorithm. Multiplication tables constitute yet another important aspect of the textbook as well as an integral part of elementary classes; it is astonishing that multiplication tables have been used as a teaching method since the 20th century, continuing into the 21st century. The only recent change is that the tables now reach multiplication by 12 (Figure 5).

ضرب جدولی — باخود کرات (مابعدی)					
۲۴	۱۲	۲	۲۰	۱۰	۲
۳۶	۱۲	۳	۳۰	۱۰	۳
۴۸	۱۲	۴	۴۰	۱۰	۴
۶۰	۱۲	۵	۵۰	۱۰	۵
۷۲	۱۲	۶	۶۰	۱۰	۶
۸۴	۱۲	۷	۷۰	۱۰	۷
۹۶	۱۲	۸	۸۰	۱۰	۸
۱۰۸	۱۲	۹	۹۰	۱۰	۹
۱۲۰	۱۲	۱۰	۱۰۰	۱۰	۱۰
۱۴۴	۱۲	۱۲			

Multiplication Table							
2	times	12	24	2	times	10	20
3	-	12	36	3	-	10	30
4	-	12	48	4	-	10	40
5	-	12	60	5	-	10	50
6	-	12	72	6	-	10	60
7	-	12	84	7	-	10	70
8	-	12	96	8	-	10	80
9	-	12	108	9	-	10	90
10	-	12	120	10	-	10	100
12	-	12	144				

Figure 5. Multiplication table.

Unlike the other three operations, the first task of the division operation was quite complex. The textbook began by asking the question "Do you think 5 fingers can divide into two parts (p.9)" which is a difficult question, as compared to questions for other basic operations, such as "If I add 3 fingers to 4 fingers, how many do I have? Or if I close 3 fingers out of 5, how many fingers are left (p. 10)?" The numbers chosen for the division question imposes difficulties for beginners. Moreover, the textbook omits to explain the division concept in the previous pages; therefore, children need to consider how they can divide five into a whole number. In addition, formalizing language is used for division. In the exercises, generally how many or half refers to the division concept. The following question is an example:

How many groups of 4 fingers are there in 8 fingers? What is half of 19 (p. 70)?

The textbook method used for the division is the same as that for the other basic operations; that is, a question-answer strategy with an inductive teaching method. The elements of division are introduced using daily language first; followed by examples, and finally the mathematical names and the division algorithm.

3.3 Theme III: The concept of measurement

Textbook analysis also revealed the existence of many measurement instruments such as time, recipe ingredients, weight, distance, and size, at an elementary level. Units of measure are converted into others, and figures support visual images and daily life examples.

Time is the first measurement concept and clock is the measurement instrument used in the textbook (Austin, Thompson, & Beckmann, 2005). Using a question-answer technique, the units of time (i.e., seconds, minutes, and hours) are introduced. The explanation of time duration is quite intriguing. Questions arose while introducing the names of the clock's hands. For instance, the textbook states: "What does the minute hand refers to?" Then, it states: "It shows minutes". Therefore, it is assumed that children know the minute concept. Youngsters might be expected to independently explore this concept. Figure 6 depicts the minutes on a clock in five-minute intervals, which is incorporated into a question.

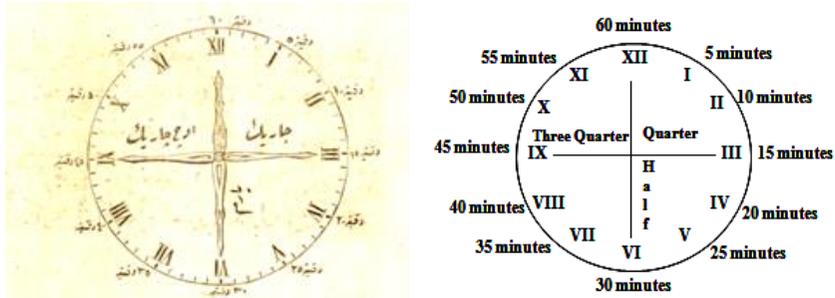


Figure 6. Time clock

The second measurement concept is the meter measurement unit, and a real life situation is presented as an example to illustrate it: “The meter is used for measuring fabric.” “What is the meter for? The meter measures the length of fabric, of a wall, or of a plank (p. 55).” Figure 7 depicts a seller using a meter stick. This might indicate that the metric system was already in daily use in the 19th century. A new law regarding the measurement system was promulgated in 1869 (Günergün, 1993). According to this law, the metric system, in addition to the unique measurement system of the state, was prescribed for length measurements, so a dual system was in use. Because of complications in enforcing this system nationwide, both systems are shown in the textbook. This, we believe, is an example of the use of the textbook as an enculturation instrument (Walker and Horsley, 2006, p.118) for reshaping the society according to the western values.



Figure 7. Measuring fabric

Units are converted according to the base-10 system, and the multiples and sub-multiples of the units follow a decimal pattern, which the textbook depicts by using the phrases bigger than and less than. Pumala and Klabunde (2005) support the idea of designing the smallest and largest units around the powers of ten. Therefore, this strategy provides opportunity for both developing students' number skills and measurement units. Moreover, the meter stick precision is decimeters (there are ten of these per meter) and centimeters (ten of these per decimeter) (Figure 8)

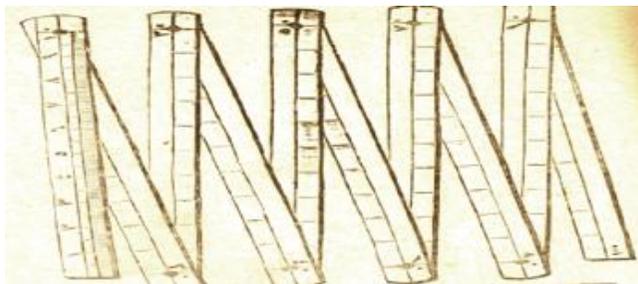


Figure 8. Meter stick

Another measurement concept is the liter. The sub-liters and multi-liters are illustrated according to the base-10 system which might facilitate pupils' place value understanding and an effective method of relating concepts with numbers (Van de Walle et al., 2010). Gram is the next measurement unit introduced in the textbook, and the figure above elucidates this measure of mass on a balance scale illustrated in Figure 9. The mass is pictorially represented in grams, kilograms, hectograms, and ten grams.

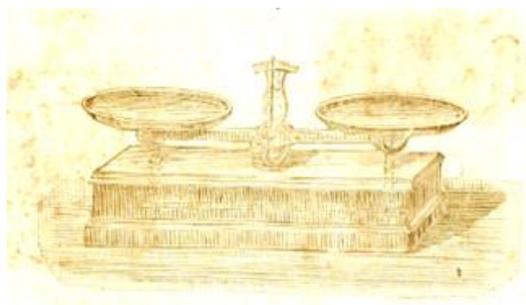


Figure 9. Scale (Terazi)

The textbook briefly presents old measurement units in addition to the new system. The old system included various kinds of units. For example, length was measured as, translates into English as, architecture's cubit (Mi'mar arshini), bazaar's cubit (charshi arshini), and ell (endaze).

3.4 Theme IV: Procedural skills and level of challenge in questions

The problems seemed to be the most interesting parts of the textbook. These included many challenging tasks and intriguing questions. There is a tendency to ask different kinds of questions before providing explanations. Considering the difficulty level of the questions, which is relatively high, the author might intentionally have used this approach. Further, the solutions require recalling previous knowledge and the ability to apply it in any given situation.

In the textbook the questions regarding number concept mostly employ writing exercises: in which memorization is encouraged implicitly or explicitly (Angliheri, 2006):

Write the numerals above many times, write the following numbers five times neatly under each other, mentally calculate, memorize it, write what is written below, and complete the blanks (p.7-38).

The numbers for the exercises were chosen in a careless way, resulting an inconsistency between the content and the questions asked; for example, although numbers from 20-30 are introduced before the question, the exercises include 71 minus 1, and 50 minus 3. Another example of a challenging task concerns addition. As illustrated in Figure 10, three single digit numbers are added, although the question follows from addition of numbers from ten to twenty. In another example, students are asked to multiply a five digit numbers by two a two digit number, without providing knowledge of how to handle such a task as shown in Figure 11.

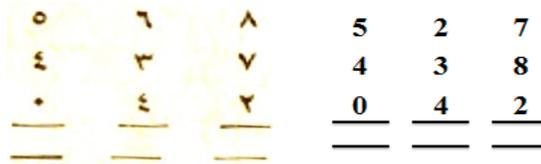


Figure 10. Addition of three numbers

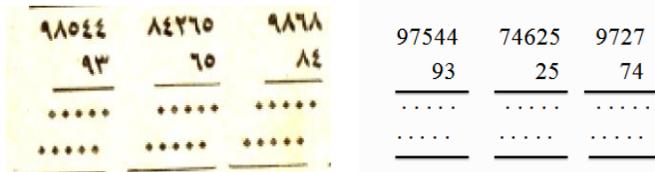


Figure 11. Multiplication with multi-digits

Problems in the textbook generally are of an identical type; and repetitive drudgework is expected for their solutions. These drill exercises help children to calculate the operations as a reflex action (Van de Walle, 2010; Anghileri, 2006). These are used in exercises for numbers and basic operations, and measurement units. Examples of drill exercises are:

1. 20 subtracted by 1; how many are left? Subtracted by 3? Subtracted by 2? Subtracted by 4? Subtracted by 5? Subtracted by 2?
2. How many liters are 5 deciliters? 4 hectoliters? 9 hectoliters? 2 hectoliters? 16 hectoliters? 10 hectoliters? 20 hectoliters? 7 hectoliters (p.19)?

There are many mental calculation questions such as "How many ones and tens are there in 56? In 42? In 63? In 68? In 70? In 48? In 26? In 50? In 16? In 18? In 13? In 15? In 3(p. 22)?" Moreover, the result unknown strategy with real life contexts is used in the addition and subtraction problems (Van de Walle et al., 2010; Anghileri, 2006). The questions "What is the total of 20 books and 7 notebooks? (p.72)" and "Ahmet Bey has 10 pens, if he takes 4 more pens, how many pens does he have in the end? (p. 72)", for example, ask about the whole by indicating the parts. Subtraction problems, on the other hand, mostly ask about the part from the whole. In addition, an interesting problem type in subtraction involves combining addition and subtraction into one question:

I have 843 grains of wheat in a bag.

- 1) *How many grains of wheat are left if I take 40 grains of wheat and add 100 more? (p.52)*
- 2) *A merchant has 768 coins. He added 100 coins more. Then he took 10 coins. How many coins are left at the end? (p.76)*

The analysis indicated that multiplication and addition are used more frequently than the other two operations. However, in the arithmetic problems section, the author emphasized number sense skills and addition much more than the other concepts.

4. DISCUSSION

The findings of this study indicate that the textbook uses an inductive method. The general approach is self-regulating, direct, but at times too challenging. The textbook provided opportunities to analyze number relationship tasks using real life connections and multiple representations yet procedural knowledge was fostered heavily. Thus, reasoning was not emphasized. There were two major findings in this study which will be discussed in this section in reference to the literature: First one is the evidence of a progression for multiple representations and real-life examples, which is compatible with the developmental level of students. Secondly, the textbook focused too much on procedural skills and ignored problem solving or reasoning skills. As a result, the discussion of the findings organized under three headings as below.

4.1 Findings related to multiple representations

Using multiple representations is accepted as core methodology in mathematics education, and interacting with multiple representations promotes deeper understanding and elucidates meanings more effectively (Elia, Gagatsis, & Demetriou, 2007). All topics in the textbook are related to real life issues and illustrate connections using the appropriate mathematical representations, which increases the flexibility of learning (Brahier, 2013). When situations are represented in various different ways, young learners will be able to independently explore and choose the most appropriate method for themselves (Fuson, 1986). Explaining basic operations and numbers using words, pictures, and images can help children in effective thinking and learning (Van de Walle et al., 2010).

Real-life word problems support the ideas conveyed by these findings. A highly integrated understanding can be achieved when contextual problems are involved (Carpenter, Hiebert, & Moser, 1999). Asking word problems with larger numbers can elucidate the operations (Jung, Kloosterman, & McMullen, 2007). Real life problems may capture learners' interest, allowing to develop a positive disposition toward mathematics (Hanna, 2000).

The representation of measurement units in the textbook is consistent with extant studies. (Thompson & Preston, 2004). There is evidence that answering with units in measurement problems (Lehrer, 2003) can help mitigate learners' misconceptions. Measurement activities help young learners to connect their ideas regarding numbers to the real world and provide them with number sense; further, using multiples of ten to show relationships among nonstandard measurements develops the idea of the base-10 system of numeration (Hiebert, 2013).

The findings in relation to language usage can be explained by using Pimm's study (1987) on linguistics: using several semiotic systems simultaneously (symbols, oral language, written language, and visual representations) can help learners in recognizing mathematical concepts easily. According to Anghileri (2006) formalizing language is beneficial during the first stage of introducing mathematical concepts. Specifically, using the expressions and words develops children's understanding: one more, two more, ten less, altogether, and leaves (Anghileri, 2006). Besides, mathematical symbolism and written language sufficiently help in constructing the meaning. In this context, traditional phrases such as copy, write, memorize, and drill require lower level thinking abilities, and these words do not sufficiently help learners in performing mathematical operations (Van de Walle et al., 2010).

Relating different topics to each other helps young learners in visualizing the coherent whole; linking the base-10 models with the written forms of numbers can increase a child's awareness (Kari & Anderson, 2003). Establishing the relationships between operations and numbers can also help children in arriving at accurate mathematical judgments (Anghileri, 2006), and in developing useful strategies for dealing with operations and numbers (McIntosh, Reys, & Reys, 1992). Representing the relationship among addition-subtraction and multiplication-division may establish useful connections for arithmetic problems.

4.2 Findings related to procedural skills

The textbook can be understood using different keywords used throughout it (e.g., less, more, times) (Hegarty, Mayer, & Monk, 1995). The superficial usage of these words may lead young thinkers to believe that certain words will solve certain tasks (Schoenfeld, 1992). This restricted tendency in students' thinking as regards to different reasoning strategies may be one of the main causes of learning difficulties. Reasoning is established based on experiences, as problem solvers transfer and combine solution procedures from familiar situations or apply the same strategy with surface consideration (Lithner, 2000).

Challenging tasks can be determined according to the effectiveness of developing the creativity of young learners (Powell, Borge, Fioriti, Kondratieva, Koublanova, & Sukthankar, 2009). Memorization cannot help learners with mastering basic concepts, and it damages their attempts to learn

mathematics. Memorization and drill exercises might not be effective enough to justify the knowledge gained by using them. Therefore, challenging tasks can prepare students to create their own strategies. However, the task should be carefully designed because learners' experiences must be consistent with the given tasks (Schoenfeld, 1992); merely indicating in the textbook to solve the following problems without attempting to cement learners' knowledge is not an ideal approach for students' learning (Askey, 1999).

The findings with respect to the analysis can be clarified with the accumulation of knowledge of some procedures. Constructing knowledge begins at the implicit and procedural level and, then, it gradually attains the explicitly well-understood level (Karmiloff-Smith, 1995). Learning begins with the procedures and, then, an understanding of the content is robustly constructed (Rittle-Johnson & Alibali, 1999). Building conceptual knowledge is at the reflective level and requires recalling previous knowledge while reflecting on the information being connected; it is a higher level of thinking (Schoenfeld, 1992). Considering the level of the textbook, the explanations of the arithmetic concepts are at the primary level.

The first part of the procedural knowledge form of mathematics includes familiarity with symbols and awareness of the syntactic rules. The knowledge of symbols and the syntax of mathematics generally refer to surface level features; this is not a higher level of understanding. The introduction of numbers and measurement in the textbook belongs to this category. The second part comprises rules and procedures that connect the textbook with basic operations. Systematic instruction may induce learners to recall certain kinds of procedures. In this context, basic facts and arithmetic problems of the textbook can be related to the second category (Hiebert, 2013).

The accumulation of information in the early phases of performing arithmetic operations may require the execution of many procedures; this might induce learners to memorize individual pieces of information. However, the mathematics education literature tends to cite the same kinds of problems in primary school, and these problems are not well-designed in terms of supporting higher level thinking (Star, 2005; Star & Rittle-Johnson, 2008). Competent mathematical performance may not occur when these repetitive problems are continuously asked, and the understanding of procedures in two or three problems can lead the solver to think that proficiency has been achieved. Therefore, a student might erroneously believe that there is no need for more thinking (Mayer, 2002).

There is over-emphasis on drill exercises in the textbook which is a continuing trend in today's textbooks. According to Van de Walle et al. (2010), some textbooks move directly to memorization of facts after presenting concepts regarding basic operations. However, students face difficulties in the fourth and fifth grades since they have not mastered these operations. Then, in middle school, these students might still lack robust knowledge of the basic operations. This implies that memorization may not develop students' skills. Different strategies and processes for basic operations cannot be developed through drill exercises (Brownell & Chazal, 1935).

According to Carpenter and Lehrer (1999), the procedures used by young children for conceptual knowledge are limited. Therefore, procedures lack the flexibility required to maintain the conceptual framework, and problems are solved by directly modeling the given processes and representations. Young children may be unable to undo their actions or to take apart the pieces; they follow a sequence of procedures to achieve a result, and they cannot reverse it.

4.3 Implications for practice and for future research

According to the results of this study, we believe that mathematics education of Ottomans in the elementary level provides different kind of opportunities to connect mathematics with real life. Therefore, new textbook authors might provide this opportunity in the modern Turkish elementary level mathematics textbooks. In addition, there was some evidence to traditional teaching methods such as memorization, and drill exercises. Therefore, the young learners might not be encouraged to develop higher mathematical thinking in the early 20th century. The emergence of

new studies might reveal the possible aspects of mathematics education that shape our societies. In this respect, textbook writers should review the historical textbook analysis studies.

The investigation of the textbook may be informative for the enlightenment of historical mathematics education, and the results from this study add to the emerging body of literature on mathematics textbook analysis. The reported findings revealed that there was need to improve instruction in order to enhance the learning of students with disabilities. Traditional teaching methods (memorization, drill exercises) should not be more dominant than methods that foster reasoning and problem solving skills.

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