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Fifth Grade Students' Performance and Common Errors in Equivalent Fractions#

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Research Article

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ABSTRACT

This study aimed to investigate the performance and common errors of fifth grade students in equivalent fractions. The study was conducted with 435 fifth grade students from two different middle schools in Salihli, which is a district of Manisa province, in the spring semester of the 2021 – 2022 academic year. In the study, survey design, one of the descriptive research models, was used. As data collection tool, a test was developed by the researchers. Equivalent Fractions Knowledge Test, consisting of six open – ended questions, was administered to all fifth-grade students at once. Students' responses were analysed with descriptive statistical methods. According to the results, the overall performance of students in the test was low. It was observed that the students showed the highest performance in a question which included area model, and the lowest performance in a question given in context which included set model. Additionally, the most common error was that students considered multiplying a fraction by 2 and expanding it by 2 as the same algorithm while they also confused similarly for dividing and simplifying algorithms. To prevent this confusion, it can be suggested to pay attention to the use of mathematical language properly during teaching.

Keywords: Equivalent fractions, unit equivalence, fifth grade, simplifying and expanding fractions, common errors

Beşinci Sınıf Öğrencilerinin Denk Kesirler Konusundaki Performansları ve Yaygın Hataları#

Bilgi
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ÖZ

Bu araştırmada beşinci sınıf öğrencilerinin denk kesirler konusundaki performanslarının ve yaygın hatalarının belirlenmesi amaçlanmıştır. Araştırmanın örneklemini 2021 – 2022 eğitim – öğretim yılında Manisa ilinin Salihli ilçesine bağlı iki farklı ortaokulda öğrenim gören 435 beşinci sınıf öğrencisi oluşturmuştur. Araştırmada betimsel araştırma türlerinden birisi olan tarama deseni kullanılmıştır. Veri toplama aracı olarak araştırmacılar tarafından denk kesirler konusunda bir test geliştirilmiştir. Altı adet açık uçlu sorudan oluşan Denk Kesirler Bilgi Testi, tüm beşinci sınıf öğrencilerine tek seferde uygulanmıştır. Öğrencilerin teste verdikleri cevaplar betimsel istatistik yöntemleri ile analiz edilmiştir. Elde edilen bulgulara göre, öğrencilerin testteki genel performansı düşük bulunmuştur. Öğrencilerin en yüksek performansı alan modeli içeren bir soruda, en düşük performansı ise bağlam içerisinde verilen bir küme modeli sorusunda sergiledikleri görülmüştür. Ayrıca denk kesirler konusunda karşılaşılan en yaygın hata ise öğrencilerin bir kesri 2 ile çarpmayı ve 2 ile genişletmeyi aynı algoritma olarak görürken bölme ve sadeleştirme algoritmalarını da benzer şekilde karıştırmaları olmuştur. Bu karışıklığın önlenmesi için öğretim esnasındaki matematik dilinin doğru kullanılmasına dikkat edilmesi önerilebilir.

Anahtar Kelimeler: Denk kesirler, birim denkliği, beşinci sınıf, kesirleri sadeleştirme ve genişletme, yaygın hatalar

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Introduction

Fractions have an important place in mathematics education due to both the variety of meanings they contain (part - whole, measure, operator, quotient and ratio meanings) and their relation with many subjects such as decimals, percentages, ratio, proportion and rational numbers (Aksoy & Yazlik, 2017). That's why it is possible to encounter many studies about fractions, which is one of the mathematics subjects that students have difficulty in understanding, conducted in our country (Aksoy & Yazlik, 2017; Aksu, 1997; Aytekin & Toluk-Uçar, 2014; Biber, Tuna & Aktaş, 2013; Eroğlu, Camci & Tanışlı, 2019; Haser & Ubuz, 2002; Kocaoğlu & Yenilmez, 2010; Okur & Çakmak-Gürel, 2016; Özaltun, Danacı & Orbay, 2020; Pesen, 2007; Soylu & Soylu, 2005). When these studies are examined, it is seen that the subject of fractions is approached holistically, and the sub-topics of this subject are handled superficially. However, the internalization of each sub-topic in fractions has also importance for the others. At this point, understanding of equivalent fractions conceptually is accepted as a step towards a better understanding of operations with fractions (Jigyel & Afamasaga-Fuata'i, 2007). Payne (1976) also stated that the topic of equivalent fractions is necessary for all operations. In other words, in order to perform addition and subtraction operations with fractions, it should be known that equivalent fractions must be expressed in equal sized units (Ratnasari, 2018), that is, knowledge of the processes of creating equivalent fractions should be obtained. In the literature, not many studies were found on equivalent fractions. In general, it was seen that equivalent fractions were included in some of the studies in which fractions were handled holistically. This situation has necessitated further and deeper investigations towards equivalent fractions. It is believed that this study will contribute to increasing awareness of mathematics teachers about teaching the subject of equivalent fractions, and accordingly, it offers some useful pedagogical tips that they can follow in the classroom while teaching this subject.

Equivalent Fractions

Although they have different numerators and denominators, fractions which represent the same amount are called "*equivalent fractions*" (Van de Walle, Karp & Bay-Williams, 2013). There are infinitely many equivalent fractions that can be formed without changing the value of a fraction (Lamon, 2012; Pedersen & Bjerre, 2021). According to Lamon (2012), being able to form a unit fraction is the basis for understanding equivalent fractions. Equivalent fractions form the basis of ordering, addition and subtraction of fractions. In respect to this, Haser and Ubuz (2002) found that the errors made by the students while simplifying the fractions caused underperformance in four operations with fractions. It seems that equivalent fractions serve as a bridge by providing an important transition between the various concepts and operations within the subject of fractions. In this case, it has importance to internalize the concept of equivalent fractions and the processes of creating equivalent fractions in order to have a comprehensive understanding of the subject of fractions.

Big ideas about equivalent fractions

The big ideas about equivalent fractions can be listed as follows:

- The internalization of the unit fraction is the basis for understanding equivalent fractions.
- There is a multiplicative relationship between the numerators and denominators of equivalent fractions, not additive relationship.
- The set of fractions which are equivalent to a fraction has infinitely many elements. For example, $\frac{5}{6}=\frac{10}{12}=\frac{15}{18}=\frac{20}{24}=\frac{25}{30}=\frac{30}{36}=\cdots$
- It is critical for students to be able to make connections between the symbolic representation of the fraction and the representations of the area model, length model, and set model.
- It is important to make use of asymmetrical examples and non-examples as well as typical and symmetrical examples in the representation of equivalent fractions.

In accordance with current curriculum of our country (Ministry of National Education [MoNE], 2018), fractions are introduced from the first years of elementary education and many new concepts are built on this subject in the following years based on it. Especially, expanding and simplifying fractions to get equivalent representations of them are taught for the first time and only in the 5th grade. But later, these instructional objectives are used in following subjects such as ordering fractions, four operations with fractions, decimals, and percentages. Students who cannot internalize the concept of equivalent fraction, which serves as an important bridge between the mentioned subjects, cannot go beyond memorizing. For this reason, it has importance to examine the students' knowledge about equivalent fractions in depth and to reveal the current situation.

Accordingly, it is aimed to reveal the performance of 5th grade students on equivalent fractions and to determine their common errors in this study. In line with the purpose of the research, this study aims to answer the following research questions:

1) What is the performance of 5th grade students on equivalent fractions?
2) What are the common errors of 5th grade students about equivalent fractions?

Method

Research Design

Since the aim of the research was to examine the current knowledge of 5th grade students about equivalent fractions and to reveal their performance on this subject, the survey design, one of the descriptive research types, was preferred. Descriptive research refers to studies that describe a current situation as precisely and carefully as possible and ensure that this situation is revealed exactly (Büyüköztürk et al., 2020). A causal relationship is not established between the data obtained in the descriptive research type, only the co-existence relationship of these data is observed (Hocaoğlu & Akkaş-Baysal, 2019). Survey design, which is known as one of the descriptive research types, is a method that includes collecting data from as large sample as possible and presenting the findings with descriptive statistical calculations to describe the current characteristics of a group on a particular subject (Sezgin-Selçuk, 2019). In determining the common errors of students about equivalent fractions, an in-depth analysis was made, and categories were formed according to the students' responses.

Population and Sample of the Research

The target population of the research was determined as all 5th grade students in Manisa in the 2021 – 2022 academic year. The students who would constitute the sample of the research were determined among the middle schools of the Salihli district of Manisa province, where the corresponding researcher resides, by means of convenience sampling. Convenience sampling is a method in which the sample is determined in accordance with the availability, considering conditions such as time, money, and location (Büyüköztürk et al., 2020). Since the concept of equivalent fraction and the instructional objectives of getting equivalent fractions through expanding and simplifying were included for the first time and only in the 5th grade according to our curriculum (MoNE, 2018), it was deemed appropriate to select the sample from students at this grade level. The sample of the research consisted of all 5th grade students studying in two different middle schools in Salihli district of Manisa province in the 2021 – 2022 academic year. More specifically, 435 fifth grade students in these two public schools, which were affiliated to the Ministry of National Education and had a medium socio-economic status, participated in the research. It was observed that 47% of these students were female and 53% of them were male. In this case, it can be said that distribution of the students was close to each other in terms of gender.

Data Collection Tool

Considering the big ideas about equivalent fractions which are mentioned in the introduction part, a data collection tool was developed by the researchers in accordance with the level of 5th grade students. Equivalent Fractions Knowledge Test consisting of 6 open-ended questions was developed by examining related studies in the literature, mathematics course curriculum (MoNE, 2018) and 5th grade mathematics textbooks which were approved by the Ministry of National Education for using in the mathematics lessons (Durmus & Ipek, 2019; Goksuluk, 2022). The test is included in the Appendix 1.

Validity and reliability of the test

The *content validity* of the test was ensured by the opinions of the experts and the changes made in this direction. In the form prepared to apply for expert opinions, a table was created by matching the big ideas about equivalent fractions, the items developed in line with the limits of the instructional objectives in mathematics curriculum, and the resources used in the meantime. Then, this table was sent to the experts in order to get their opinions.

According to the expert opinions, it was decided to ask the students to explain in two different ways in the first question to see whether they internalized the subject of equivalent fractions or not. In addition, one part of the asymmetric area model in the third question was replaced after getting the expert opinions. Because it was believed that students who did not prefer to approach the question procedurally could group more easily with the last version of model. During the test development process, it can be said that the most changes, in terms of both context and visually, were made in the sixth question. The context of this question was primarily based on marbles. However, in the pilot study, it was observed that the set model prepared to represent the marbles was confused with the dice by the students. Besides, feedback was received from the experts that the context of the problem was not clear enough. Thus, in order to make the question more understandable, the context of the question was changed, and it was decided to use a fruit juice story as in the final version of this question.

A pilot study was carried out with 122 students randomly selected among the 6th grade students in the schools where the main data would be collected. The items were revised according to the feedback obtained from the pilot study. To reach the *internal consistency* of the test, the Cronbach-α reliability coefficient was calculated and it was obtained as 0.70. Reliability coefficients of 0.70 and above are considered sufficient for the reliability of a test (Fraenkel, Wallen & Hyun, 2011). Therefore, in the light of reliability coefficient obtained, it can be said that the applied test is reliable.

Equivalent Fractions Knowledge Test

The first question of the test was handled by Van de Walle et al. (2013). In this question, students are asked to explain whether the fractions $\frac{2}{6}$ and $\frac{1}{3}$ are equivalent to each other. In other words, students are expected to make explanations that will reveal their knowledge of an existing equivalence in this question. Thus, the related big idea for the first question is being able to explain equivalence of fractions through the relationship between the numerator and the denominator. It is predicted that this question may seem unusual to the students. Because students generally tend to create a new fraction which is equivalent to given fraction by simplifying or expanding rather than clarifying a statement which is known its truth.

The second question of the test was developed based on the study of Wong and Evans (2007). In this question, students are asked to shade $\frac{3}{3}$ of the circular area model divided into 6 equal parts, that is, shading the whole. Thus, the related big idea for the second question is being able to transition between the symbolic representation of the fraction and the circular area model representation of it. While Wong and Evans (2007) used a rectangular area model in their study, a circular area model was preferred for the same purpose in this study. The reason for this is that since a rectangular area model was included in one of the following questions, the researcher wanted to diversify the test by using a circular area model which the students are also very familiar with.

The third question of the test was developed relying on the study of Kaur and Pumadevi (2009) based on the asymmetrical shapes. In the study conducted by Kaur and Pumadevi (2009), it was concluded that mathematics textbooks, which mostly contained symmetric/typical examples and activities, were not sufficient for students to develop in-depth understanding of equivalent fractions. Since the use of rectangular and circular typical area models was preferred for teaching of fractions in also our country's math textbooks (Durmuş & İpek, 2019; Göksülük, 2022), such a question was asked, wondering how the students would perform on an asymmetrical/atypical shape. In this question, the related big idea is being able to transition between the symbolic representation of the fraction and the asymmetrical area model representation of it.

The fourth question of the test was developed based on the study of Wong and Evans (2007). In this question, students are asked how many more parts need to be painted in order to have $\frac{6}{14}$ of a rectangular area model, which is divided into 7 equal parts and $\frac{1}{7}$ of it is painted. Thus, the related big idea for the fourth question is being able to transition between the symbolic representation of the fraction and the rectangular area model representation of it. Due to the nature of getting equivalent fractions, students should be able to make a correct transition between different units in this question as well. Unlike the area models in the second and third questions, it is necessary to get smaller units in this question, not bigger ones. Thus, while the students were given opportunity to visualize the simplifying a fraction by 2 in the second and third questions, they were given the opportunity to visualize the expanding a fraction by 2 with this question.

The fifth question of the test, which includes a length model, was developed by the researchers. As a result of the literature review, there was no example of a length model which was appropriate for the purpose of the study. However, in order to benefit from representations of fractions other than symbolic and area ones, this question was developed by the researchers. For this purpose, an image of ruler with marked its midpoint was given and students were expected to interpret the equivalence of fractions through length model at this time. In other words, the point A is placed in the middle of a ruler which is divided into 24 equal parts. And then, students need to compare the distance of $\frac{5}{6}$ of the left part of point A and the distance of $\frac{20}{24}$ of the right part of point A. Thus, the related big idea for the fifth question is being able to transition between the symbolic representation of the fraction and the length model representation of it.

The sixth question of the test, which includes a set model, was also developed by the researchers. In this question, the related big idea is being able to transition between the symbolic representation of the fraction and the set model representation of it. For this purpose, a context that required working on the set model was needed first. As in the length model, a set model context has also not been found as a result of the literature review, so this last question of the test was developed by the researchers. The context of it includes 24 cans of juices in total and 16 of them are cans of cherry juices while 8 of them are cans of apricot juices. These cans of juices are divided into 6 boxes, with the same type of juices together, thus each boxes contains 4 cherry juices or apricot juices. In this question, students are expected to rearrange these boxes to contain the same type of juices but with a different number of them. As a result, students are expected to express how many of the boxes containing apricot juices are in all boxes, with any two of the fractions $\frac{1}{3}$, $\frac{4}{12}$, and $\frac{8}{24}$.

Data Collection Process

The data collection process was carried out in April with 221 students in a school and in May with 214 students in the other school. In this process, which was planned considering the schedules of the teachers at the schools, the data were collected at once in a 40-minute class hour by using paper-pencil. It was observed that this duration given for answering the test was sufficient during both the pilot and the main studies.

The data were collected by the corresponding researcher under the supervision of the mathematics teachers at schools. In the meantime, the students were briefly informed about the identity of the researcher, the subject of the test and the duration of the test. In addition, it was stated that this test would not be scored in any way and would not affect the students' mathematics course scores at school.

Internal and external validity of the research

The *internal validity* of the study was ensured both by selecting classes neutrally in the pilot study and including all students in two schools without choosing among the classes in the main study. Also, internal validity of the study was tried to be controlled by collecting data at once from the students who had similar experiences in the very similar classroom environments of two public schools. The researchers avoided conducting this study on any special dates which included various events and celebrations in schools. Otherwise, the students' answers could be affected by external factors, and this could lead to a decrease in internal validity. In addition, the study was carried out under the control of corresponding researcher in order to minimize the effect of the interaction, that might occur between the students during the study, on the results. Furthermore, it was aimed to minimize the effect of losing participants on the results by starting the research with as many participants as possible, thus the researchers attempted to control the internal validity.

In descriptive studies, it is recommended that the sample consists of at least 100 participants (Fraenkel et al., 2011). *External validity*, which is described as the degree of generalizability of the results to the population, was ensured by selecting a sample that was about 4 times larger than the minimum size for descriptive studies. Thus, the results obtained can be generalized to public schools in our country, which have a medium socio-economic status and prefer to use mathematics textbooks approved by the Ministry of Natioal Education [MoNE] in the mathematics lessons.

Data Analysis

To evaluate the performance of the Equivalent Fractions Knowledge Test, students who answered the questions incorrectly or left blank were coded as 0, and students who answered correctly were coded as 1. Then, descriptive analysis was performed by creating frequency and percentage tables.

To determine the common errors encountered in the subject of equivalent fractions, an inductive analysis, which includes the discovery of categories by examining the findings obtained in the study, was carried out by the researchers. In this respect, the errors encountered in the study were categorized and naming was created by considering the errors in the literature (Aksoy & Yazlik, 2017; Biber, Tuna & Aktaş, 2013; Hansen et al., 2016; Kocaoğlu & Yenilmez, 2010; Lestiana, Rejeki & Setyawan, 2016; Okur & Çakmak-Gürel, 2016; Özaltun, Danacı & Orbay, 2020; Pesen, 2007; Ratnasari, 2018). Thus, the students' errors were categorized under nine categories.

Results

Performance of Fifth Grade Students on Equivalent Fractions

Descriptive statistics such as mean, standard deviation, minimum and maximum values obtained from the overall test are given in Table 1.

According to the data in Table 1, the mean of the total scores of 435 fifth grade students on the Equivalent Fractions Knowledge Test was 2.40 while the standard deviation was 2.02. Since the total score that can be taken from the overall test is 6, the mean value shows that performance of the students was lower than 50%. When the total scores obtained from the test were examined, it was seen that 24.6% of the students could not answer any question correctly and they got 0 points. It was found that only 7.6% of the students got 6 points by answering all the questions correctly. According to these data, it can be concluded that the students underperformed in the test that measured the knowledge of equivalent fractions.

According to the correct and incorrect responses of students, the frequency and percentage values are given in the Table 2.

When the data in Table 2 is examined, it is seen that the highest performance belongs to the second question, which requires being able to transition between the symbolic representation of the fraction and the circular area model representation of it, with a correct answer rate of 49.2%. It is understood that the lowest performance belongs to the sixth question, which only 26.9% of the students could answer correctly. This question, in which the students showed the lowest performance, was prepared in a context to observe their ability to transition between the symbolic representation of the fraction and the set model representation of it. In addition to this differentiation in the fraction models included in the questions, the fact that the sixth question was given in a context may have caused a lower performance in this question. After the second question, it was noteworthy that the question with the highest performance was the third question, which included an asymmetric area model, with a correct answer rate of 47.1%. It was thought that the students might underperform in third question before to get results of this study, since it was seen that typical or symmetrical area models in fractions were frequently included but asymmetric ones were not included in the 5th grade mathematics textbooks examined (Durmuş & İpek, 2019; Göksülük, 2022). For this reason, it was surprising that the students showed the highest performance in this question after the second question.

Fifth Grade Students' Common Errors About Equivalent Fractions

In this research, the students' incorrect approaches were examined, and the errors encountered about equivalent fractions were collected under the following headings

Table 1. Descriptive Statistics Values Obtained From the Overall Test

	N	Minimum			Maximum			Mean	Standard deviation
		Score	f	%	Score	f	%		
Total score	435	0	107	24.6	6	33	7.6	2.40	2.02

Table 2. Distribution of the Correct and Incorrect Responses of Students

	Item 1		Item 2		Item 3		Item 4		Item 5		Item 6	
	f	%	f	%	f	%	f	%	f	%	f	%
Correct answers	201	46.2	214	49.2	205	47.1	152	34.9	156	35.9	117	26.9
Incorrect answers	234	53.8	221	50.8	230	52.9	283	65.1	279	64.1	318	73.1
Total	435	100	435	100	435	100	435	100	435	100	435	100

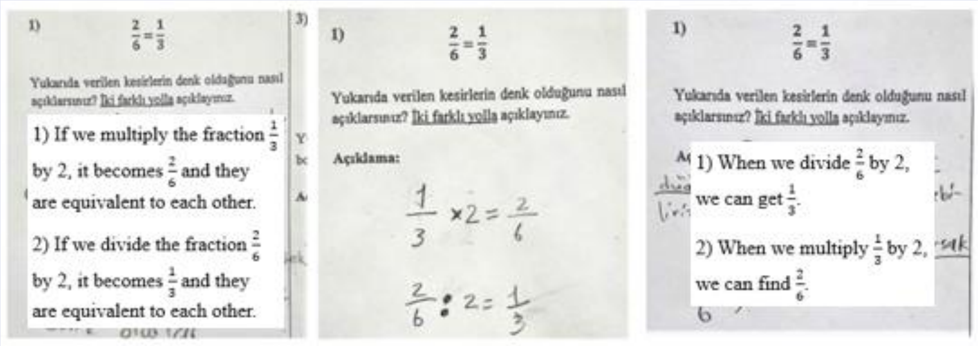

Figure 1. Considering Simplifying and Expanding as Division and Multiplication Algorithms

Considering simplifying and expanding procedures as division and multiplication algorithms. When the incorrect approaches of the students were examined, it was seen that most of the students considered multiplying a fraction by 2 and expanding it by 2 as the same algorithm while they also thought similarly dividing a fraction by 2 and simplifying it by 2 as the same algorithm. Whereas only the numerator or only the denominator of a fraction is affected in multiplication or division algorithms, there is an effect on both the numerator and the denominator of a fraction in simplifying and expanding procedures which are ways to get equivalent fractions. However, such confusion may arise when students apply the algorithms by rote, that is, without understanding. Examples of the difficulties experienced by the students in this regard are given in Figure 1. As it can be understood from these examples, correct notations and expressions could not be used even if the same operations were applied on both the numerator and denominator during the simplifying and expanding to get equivalent fractions by students. Although this type of error arose overall the test, it can be said that it was mostly encountered in the explanations of the first question.

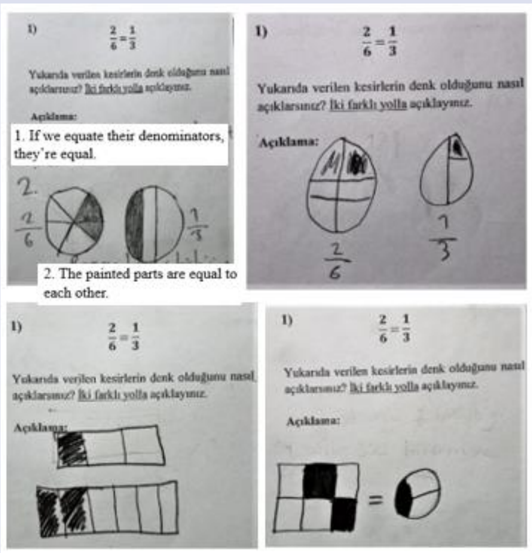

Figure 2. Inability to Form Equal Wholes For Area Models & Inability to Divide a Whole Into Equal Parts

Inability to form equal wholes for area models & Inability to divide a whole into equal parts

When students' wrong approaches were examined, it was found that there were students who had difficulties due to not being able to draw the wholes equally for area models or not being able to divide a whole into equal parts. It can be said that this erroneous approach was encountered especially in the explanations of the first question. Although it was seen that students experienced this difficulty both in drawing a rectangular and circular area model, it is possible to say that this error was more common in drawing circular ones. In other words, the rate of correct answers was higher among students who preferred to use the rectangular area model while the rate of incorrect answers was higher among students who preferred to use the circular area model to explain that two fractions were equivalent to each other. This difficulty experienced by the students is clearly seen in Figure 2.

Focusing directly on the numerator of a fraction in cases where creating equivalent fractions is required Another wrong approach of the students was to focus directly on the numerator of a fraction without considering the necessity of the simplifying and expanding procedures in cases where getting equivalent fractions was required. In other words, most of the students who answered the questions incorrectly think that the number of parts in the figure should be colored directly according to the numerator, regardless of the denominator. Examples of the difficulties experienced by students in this regard are presented in Figure 3. For example, most of the students who gave the wrong answer to the second question stated that they only painted 3 parts of the model because the numerator of the fraction included the number of 3. In other words, in this question where the entire shape must be colored, it can be seen that the students focused directly on the numbers and did not realize that by painting 3 parts, they actually colored half of the shape. Similarly, in the solutions of the third question, it was found that many students stated that they painted 4 parts of the model directly instead of multiplying both the numerator and denominator by 2 since the number of 4 is in the numerator of the fraction.

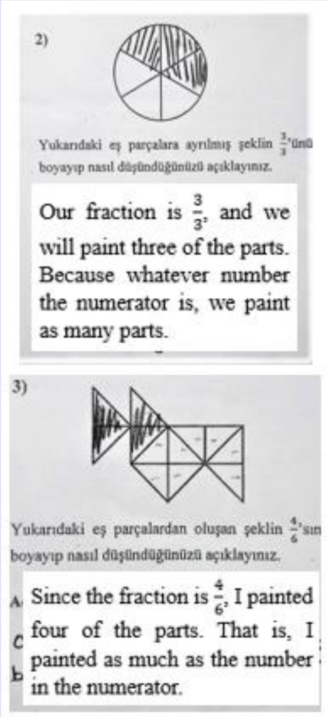

Figure 3. Focusing Directly on the Numerators of Fractions in Cases Where Expansion is Required

Inadequate internalization of whole, half and quarter in fractions

Another erroneous approach is thought to be due to the students' inadequate internalization of whole, half and quarter in fractions yet. In Figure 4, examples of the difficulties experienced by the students in clarifying the question due to such a confusion of concepts are given. In the first of these examples, the student stated that the fractions $\frac{2}{6}$ and $\frac{1}{3}$ corresponded to quarters in the explanation s/he made, even though he made a correct drawing for the solution of the first question. In the second example, it is thought that the student may have had difficulties due to the inability to internalize the concept of halves in fractions. In other words, it is seen that the student painted only half of the figure, thinking that the fraction $\frac{3}{3}$ is half of $\frac{6}{6}$ in the second question, which was required to paint the whole figure. In the last example in Figure 4, there is an approach that the student tries to reach a conclusion based on the concept of half for the solution of the third question. This student adopted the right approach by stating that $\frac{4}{6}$ of the figure was $\frac{1}{6}$ more than half of the figure, but s/he could not determine the right amount to be painted on the figure and painted less than half of the figure.

Establishing additive relationship between the numerators and denominators of equivalent fractions. Another wrong approach was that some students established an additive relationship between the numerators and denominators of equivalent fractions. One of the big ideas about equivalent fractions was that the procedures of getting equivalent fractions involved a multiplicative relationship. Moreover, understanding the multiplicative relationship, which forms the basis of the equivalent fraction procedures, is also important for the development of proportional thinking (Hansen et al., 2016). The fact that this big idea was not sufficiently internalized by the students may have caused erroneous approaches as in Figure 5. For example, based on the difference between the numerator and denominator of $\frac{4}{6}$ in the third question, it is seen that the student considered how many pieces of the 12 pieces in the given model should be painted to get same difference and consequently s/he decided to paint 10 pieces of it. That is, according to the student who made this error, $\frac{4}{6}$ and $\frac{10}{12}$ were equivalent to each other because the numerator of both fractions was 2 less than the denominator. In fact, it was observed that the same student approached the fifth question with a similar error. In this question, which includes comparing the distances on the number line, the student stated that the numerator of $\frac{5}{6}$ was 1 less than the denominator and s/he also stated that the numerator of $\frac{20}{24}$ was 4 less than the denominator. Thus, s/he concluded that the point C was located at a farther point than B.

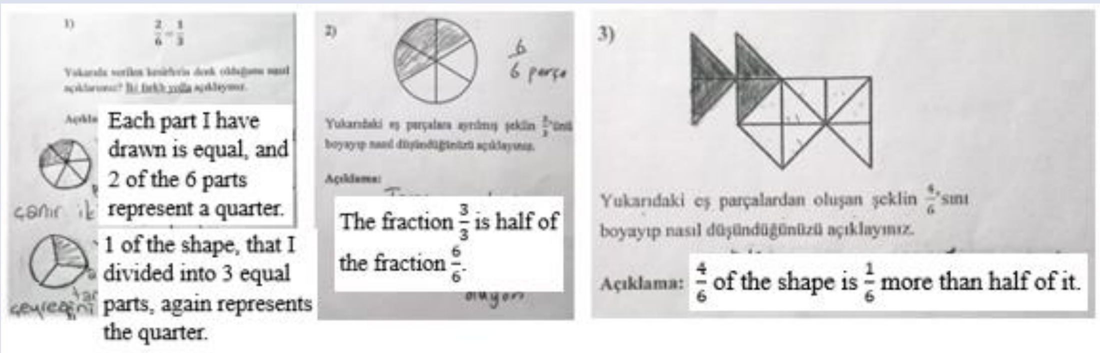

Figure 4. Inadequate Internalization of Whole, Half and Quarter in Fractions

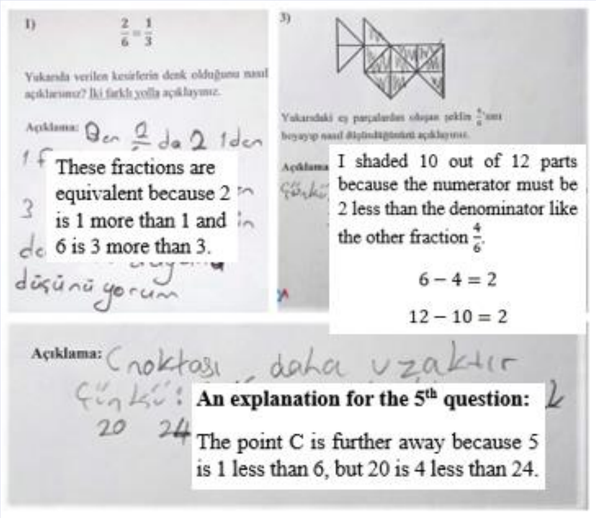

Figure 5. Establishing additive relationship between numerators and denominators of equivalent fractions

Difficulties in transition between units while forming equivalent fractions

It is thought that one of the errors encountered was due to the inability to ensure a correct transition between units while creating equivalent fractions. An example of this situation was that students did not pay attention to the fact that the area model given in the fourth question and the fraction given symbolically had different units. Therefore, they applied directly mathematical algorithms without using simplifying and expanding procedures for transition between different units. These students thought that directly 6 pieces of the given model should be painted, regardless of which unit the fraction $\frac{6}{14}$ consists of. And then, they stated that 5 more pieces should be painted by subtracting 1 from 6, since 1 piece of the model appeared painted even if unit of the given area model was $\frac{1}{7}$. There were students who had difficulties in the transition between units during their operations, as well as students who had this difficulty during their drawings. Although these students followed a correct process while trying to reach the result by making smaller the units, it was seen that they decided on the amount to be painted in the last stage based on the $\frac{1}{14}$ they formed instead of the $\frac{1}{7}$. Therefore, students who made such an error during the transition between units thought that 4 pieces should be painted instead of 2 pieces. Examples of these wrong approaches of the students are given in Figure 6.

Among the students who couldn't make a correct transition between the units in the fifth question, which included the length model, S77's approach draws attention. S77 preferred to approach the question over the parts that would remain at the ends of the ruler, unlike the other students. Although the procedures followed by the student were correct, it was seen that student couldn't make the correct transition between different units while determining the positions of the B and C points, so s/he concluded that the positions of these two points from the point A were not equal. As seen in Figure 7, firstly S77 subtracted the fraction $\frac{5}{6}$ from the left part considered as one whole, and then s/he found that a $\frac{1}{6}$ piece would remain at the left end of the ruler. However, while deciding on the position of point B on the ruler, the student assumed that the left part was divided into 6 parts instead of 12 parts, and s/he placed it at the point 1 unit inside from the left end. Then, similarly, S77 subtracted the fraction $\frac{20}{24}$ from the right part considered as one whole 1, and then s/he found that a $\frac{4}{24}$ piece would remain at the right end of the ruler. However, with a similar wrong approach, while deciding on the position of point C on the ruler, the student assumed that the right part was divided into 24 parts instead of 12 parts, and s/he placed it at the point 4 units inside from the right end. Thus, the student concluded that point C was closer to point A because of this difficulty in transition between units. However, if the student had compared the symbolic representation of these remaining parts and s/he had realized that the remaining parts of the same whole were actually equivalent to each other, he could query this wrong decision.

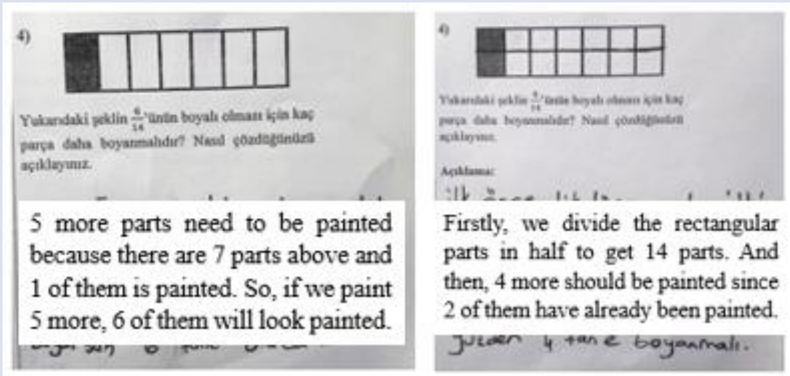

Figure 6. Difficulties in Transition Between Units While Forming Equivalent Fractions

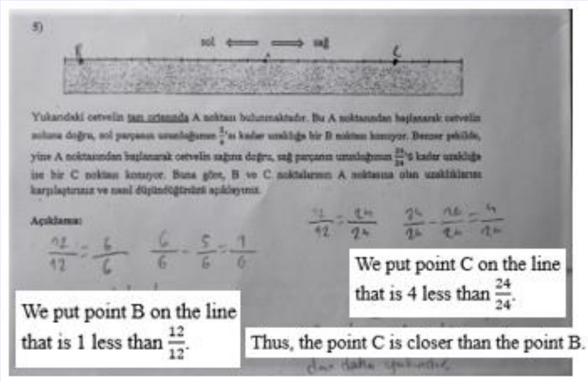

Figure 7. The Difficulty Experienced by S77 During the Transition Between Units

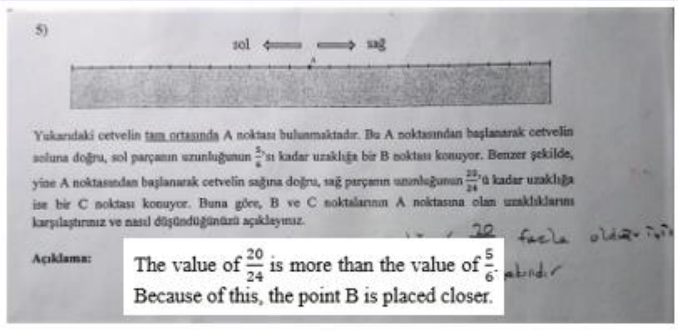

Figure 8. Comparison of equivalent fractions like whole numbers

Comparison of equivalent fractions like whole numbers

In another remarkable error specifically in the fifth question, it was observed that the students had a conceptual difficulty while comparing the fractions even if they were equivalent to each other. In other words, when comparing the fractions $\frac{5}{6}$ and $\frac{20}{24}$ in the fifth question, it was seen that these students thought like the comparison of whole numbers. For students who had this thought, the numerator and denominator were interpreted as separate entities instead of as part of a fraction. In other words, these students thought that the value of $\frac{5}{6}$ was less because it contained smaller numbers, and the value of $\frac{20}{24}$ was more because it contained larger numbers, so s/he concluded that point B was closer due to this wrong comparison. This difficulty experienced by the students is given in Figure 8.

Directly simplifying or expanding a fraction without considering context of the given problem

An error encountered specifically in the sixth question was that after the set model given in the question was expressed symbolically as $\frac{2}{6}$, the students thought that they could expand it by *any* non–zero number. However, the fraction $\frac{2}{6}$ can either be simplified by 2 or expanded by 2 and 4 due to the context of the question. Otherwise, the condition of having the same type of object in each set will not be met. It is thought that this error may have arisen from not thinking enough about the context given in the question and not being able to make sense of the question by the students. A student example about this approach is given in Figure 9. Considering this student approach given in Figure 9, it is pleasing to see that a big idea about equivalent fractions has actually developed for the student. This big idea is that the set of equivalent fractions has infinitely many elements. It can be understood from the explanation of student that s/he was aware that infinitely many equivalent fractions could be created by expanding the fraction $\frac{2}{6}$, but s/he limited her/his solution to only two different expansion operations since two different ways were requested in this question. However, the student's error here was to expand the fraction $\frac{2}{6}$ by 2 and 3 unquestioningly. While the context of this question is appropriate for expanding the fraction $\frac{2}{6}$ by 2, it is not suitable for expanding it by 3. Because the number 18, which is formed in the denominator as a result of expanding by 3, will give the total number of boxes, but 24 objects cannot be shared to 18 boxes equally. Or, similarly, the number 6, which is formed in the numerator as a result of expanding by 3, will give the number of boxes containing apricot juices, but 8 objects cannot be shared to 6 boxes equally since there are only 8 apricot juices in total. The fact that the student cannot carry out this reasoning and thinks that s/he can use directly every number for expanding procedure shows that s/he cannot go beyond memorization.

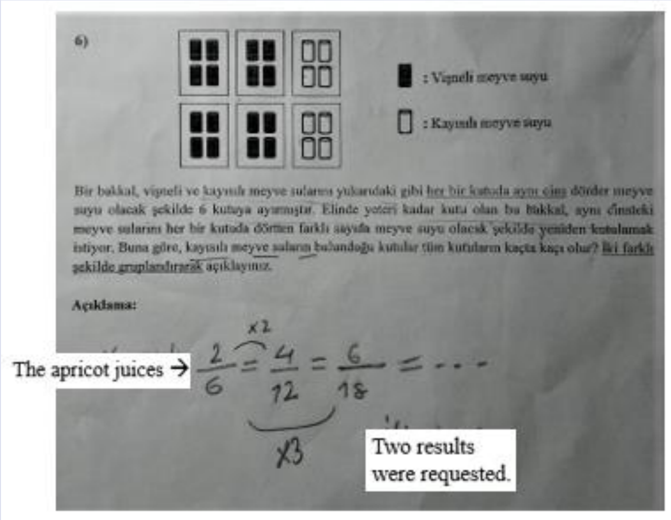

Figure 9. Directly Expanding the Given Fraction by a Fraction Which is Not Appropriate For the Context

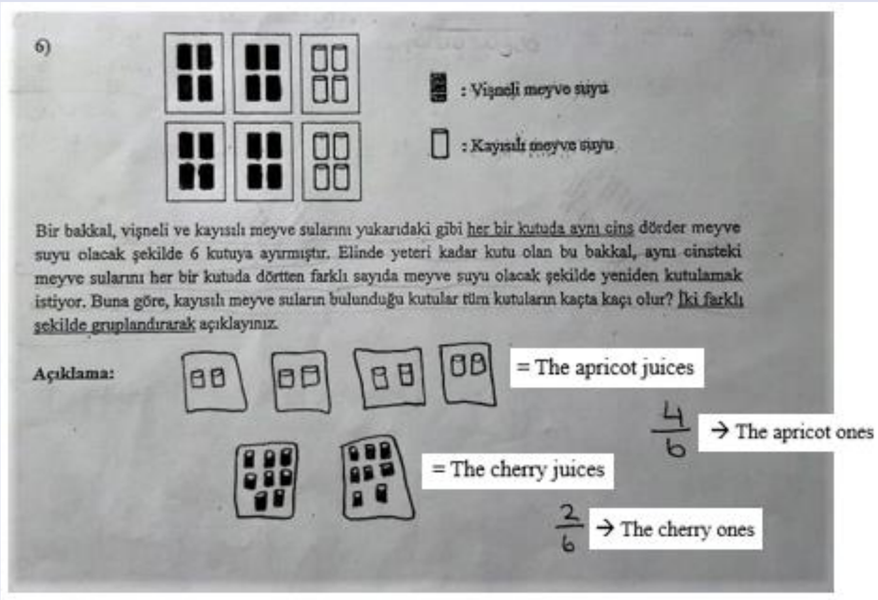

Figure 10. Inability to Create Equal Sized Units in the Set Models

Inability to Create Equal Sized Units in The Set Models

Another error specific to the sixth question was encountered while the students were creating new units through the set model. These students, ignoring that each new set should contain an equal number of objects, grouped them so that there were different numbers of cherry and apricot juices in the sets. For example, when considering a student's approach in Figure 10, it is seen that apricot juices were grouped in pairs while cherry juices were grouped in eight. This misarrangement of the set model also resulted in an incorrect symbolic representation. It is thought that the students who take this approach may not have understood yet that every new unit to be created in the set model must be equal sized as in the area and length model.

Discussion and Conclusion

According to the findings obtained in the study, it was observed that the overall performance in the Equivalent Fractions Knowledge Test was low. Similarly, it is seen that Haser and Ubuz (2002) also draw attention to students' low performance about the equivalence of fractions in their study. Furthermore, Aksoy and Yazlik (2017) also found that the lowest success rate with 38% among 105 fifth grade students in their study, in which students' errors in fractions were determined, belonged to getting equivalent fractions. In the current study, it was observed that the students showed the highest performance in the second question, which included an area model, and the lowest performance in the sixth question, which included a set model. In the 5th grade mathematics textbooks approved by the Ministry of National Education (Durmuş & İpek, 2019; Göksülük, 2022), it is seen that the representations of the equivalent fractions with the area model are frequently included while the representations of it with the set model are not included. Thus, while students have enough experience with the area model representations on equivalent fractions, they often do not have experience with the set model representations. It is thought that this situation may cause students to perform higher in the second question and lower in the sixth question.

A trend was noted in the area model preference of the students in this study, and it was observed that they made correct drawings by mostly choosing a rectangular area drawing in the area model. It was observed that there were fewer students who tried to explain by drawing a circular area, and only a few of these students were able to make correct drawings. In other words, it was seen that students who preferred to use the rectangular area model to explain the equivalence of two fractions had a higher rate of reaching the correct answer, while those who preferred to use the circular area model had a higher rate of reaching the incorrect answer. It can be said that these findings were quite similar to the findings of the study conducted by Pesen (2007). The reason for this difference in performance between area models may be that students have more difficulty to divide a circular area model into equal parts than a rectangular area model. As a matter of fact, in the study conducted by Eroğlu, Camci, and Tanışlı (2019) to develop a hypothetical learning trajectory for addition and subtraction in fractions, it was observed that sixth grade students had quite difficulty in forming equal parts on the circular area model. In this direction, it is recommended that students should work with the circular area model after they have mastered other area models like rectangular ones, and it is suggested to proceed to the odd number of divisions after the even number of them when dividing the circular area model into the equal parts (Eroğlu et al., 2019).

Another finding of the research was that some of the students preferred to start the solution of fourth question with simplifying procedure when the others preferred to start it with expanding procedure, so this decision affected their reaching the correct answer. It was determined that the rate of reaching the correct answer was higher for the students who first started the solution by simplifying the fraction $\frac{6}{14}$ with 2 and then continued with $\frac{1}{7}$ units. It was observed that most of those, who started the solution of the problem by expanding the fraction $\frac{1}{7}$ with 2, ignored that the given area model had $\frac{1}{7}$ units. That is, these students forgot that they had to simplify the fraction they obtained in the last stage by 2 again, so they made inferences over $\frac{1}{14}$ units. It is thought that this differentiation may have been encountered since the solution of the problem involves the use of at least two different algorithms which are subtraction of fractions and getting equivalent fraction.

When the findings obtained in the study were evaluated, it was observed that the students had difficulty in making sense of the fifth and sixth questions compared to the first four questions, and they expressed themselves more difficult in these last two questions. Although it is very useful to include various models such as area, length, and set models in the teaching of fractions and equivalence of them (Hansen et al., 2016; Van de Walle et al., 2013), it is obvious that students are more familiar with the area model and have more difficulties in other models. In addition, another reason why students have difficulty in making sense of the fifth and sixth questions may be that these questions are given in a context.

The most common error observed in the study was that the students considered multiplying a fraction by 2 and expanding it by 2 as the same algorithm while similarly dividing a fraction by 2 and simplifying it by 2 were the same algorithm. However, *both* the numerator *and* denominator are affected in simplifying and expanding procedures to get equivalent fractions while *just* the numerator *or* denominator are affected in the multiplication and division algorithms. It is possible to encounter this type of error in the study of Lenz et al. (2022). In the study of Lenz et al. (2022), in which the errors made by students about equivalent fractions were analyzed, it was seen that many students with low conceptual knowledge divided directly the fraction to 2 that was asked to be simplified by 2. Incorrect or inadequate use of mathematical language during teaching the concept of equivalent fractions and ways of creating equivalent fractions may cause students to make this error.

In conclusion, as a result of the findings obtained in the research, it is necessary to mention main points about the concept of equivalent fraction and the getting equivalent fractions. Undoubtedly, the concept of unit comes first because being able to create a unit fraction is the basis for understanding equivalent fractions (Lamon, 2012). In this study, especially in the fourth question, which includes the use of more than one algorithm, it was found that the students ignored the difference between units before applying mathematical operations, and as a result, they could not make the transition between different units correctly. In addition, according to the findings, it should be one of the main understandings that students should gain, that the whole should not be changed while creating new units. Finally, the students did not think enough about the change in the size and amount of the units when applying equivalent fraction procedures.

As the limitations of the research, it can be said that the sample of the research was limited to 5th grade students studying in two different secondary schools in Salihli, which is a district of Manisa province, in the 2021-2022 academic year. Additionally, when the literature was examined, it was seen that Pedersen and Bjerre (2021) discussed the concept of equivalent fraction in two conceptual aspects which are unit equivalence and proportional equivalence. In the mathematics curriculum (MoNE,2018), the concept of ratio is included for the first time at the 6th grade, and the concept of proportion is included for the first time at the 7th grade. However, since the sample of this study consisted of only 5th grade students, the concept of equivalent fraction used in this study was limited with unit equivalence, which included the meaning of part-whole.

Implications

As a result of the findings obtained, it was seen that the students were more successful in using the symbolic representation of fractions, in the representations of rectangular and circular area models, and in situations involving typical and symmetrical examples. Since this performance of the students may be due to the experiences they have gained, it is recommended to give them the opportunity to experience more with the length model, set model and asymmetrical examples. Thus, it is believed that students' understanding of the concept of equivalent fractions will be strengthened by diversifying the forms of representation to be used during teaching. In addition, it would be another suggestion to include the use of length and set models as well as area models in the representation of equivalent fractions in textbooks that serve as a guide for teachers.

Finally, it was observed that most of the students considered multiplying a fraction by 2 and expanding it by 2 as the same algorithm, while similarly dividing a fraction by 2 and simplifying it by 2 were the same algorithm. In order to prevent this confusion experienced by students, it can be suggested that teachers should pay attention to the use of mathematical language during teaching. So, in the process of introducing or teaching simplifying and expanding operations, teachers can avoid using the expressions “multiply the fraction by 2” or “divide the fraction by 2”, emphasizing that the same operation is applied to *both* the numerator and the denominator.

Genişletilmiş Özet

Giriş

Öğrencilerin kavramada güçlük çektiği matematik konularından birisi olan kesirler konusunda ülkemizde yapılmış birçok çalışma incelendiğinde (Aksoy & Yazlik, 2017; Aksu, 1997; Aytekin & Toluk-Uçar, 2014; Biber, Tuna & Aktaş, 2013; Eroğlu, Camci & Tanışlı, 2019; Haser & Ubuz, 2002; Kocaoğlu & Yenilmez, 2010; Okur & Çakmak-Gürel, 2016; Özaltun, Danacı & Orbay, 2020; Pesen, 2007; Soylu & Soylu, 2005) kesirler konusuna bütüncül bir şekilde yaklaşılarak alt kazanımların yüzeysel olarak ele alındığı görülmektedir. Halbuki kesirler konusunun içerisinde yer alan her bir alt başlığın içselleştirilmesi bir diğeri için de önem arz etmektedir. Örneğin, kesirlerle toplama ve çıkarma işlemlerinin yapılabilmesi için öncelikle bu kesirlerin eş büyüklükteki birimler cinsinden ifade edilmesi gerektiği bilinmeli yani denk kesir oluşturma süreçleri hakkında bilgi sahibi olunmalıdır (Eroğlu, Camci ve Tanışlı, 2019).

Güncel öğretim programımızda (MEB, 2018) kesirler, ilköğretimin ilk yıllarından itibaren tanıtılmaya başlanmakta ve ilerleyen yıllarda bu konu temel alınarak birçok yeni kavram kesirler üzerine inşa edilmektedir. Denk kesir kavramı ve genişletme – sadeleştirme yoluyla denk kesir oluşturma, ilk kez ve sadece 5. sınıfta öğretilmekte fakat sonrasında bu bilgilerden kesirlerin sıralanması, kesirlerle dört işlem yapılması, ondalık gösterimler ve yüzdeler gibi birçok alanda yararlanılmaktadır. Öyle ki konular arasında önemli bir köprü görevi gören denk kesir kavramını içselleştiremeyen öğrenciler ezbere işlem yapmanın ötesine geçememektedir. Bu nedenle öğrencilerin denk kesirler konusunda sahip oldukları bilgilerin derinlemesine incelenip mevcut durumun ortaya konması önem arz etmektedir.

Bu doğrultuda, bu çalışma ile 5. sınıf öğrencilerinin denk kesirler konusundaki performanslarının ortaya konması ve yaygın hatalarının belirlenmesi amaçlanmıştır. Araştırmanın amacı doğrultusunda şu problemlere yanıt aranmıştır:

1) 5. sınıf öğrencilerinin denk kesirler konusundaki performansları ne durumdadır?
2) 5. sınıf öğrencilerinin denk kesirler konusunda yaptıkları yaygın hatalar nelerdir?

Yöntem

Araştırmada 5. sınıf öğrencilerinin denk kesirler ile ilgili var olan bilgilerinin incelenerek bu konudaki performanslarının ortaya konması amaçlandığından betimsel araştırma türlerinden tarama deseninin kullanılması tercih edilmiştir.

Araştırmanın örneklemini 2021–2022 eğitim–öğretim yılında Manisa ilinin Salihli ilçesine bağlı ve araştırmacının ulaşabildiği iki farklı ortaokulda öğrenim gören 435 beşinci sınıf öğrencisi oluşturmuştur. Araştırmada denk kesirler konusundaki temel fikirler göz önünde bulundurularak araştırmacılar tarafından toplam 6 adet açık uçlu sorudan oluşan bir veri toplama aracı geliştirilmiştir.

Geliştirilen Denk Kesirler Bilgi Testine ilişkin performans değerlendirmesinin yapılabilmesi amacıyla öncelikle soruları yanlış cevaplayanlar ya da boş bırakanlar 0, doğru cevaplayanlar ise 1 olacak şekilde kodlanmıştır. Ardından frekans ve yüzde tabloları oluşturularak betimsel analiz yapılmıştır. Denk kesirler konusunda karşılaşılan yaygın hataların belirlenmesi amacıyla ise araştırmacı tarafından çalışmada elde edilen bulguların derinlemesine incelenerek kategorilerin keşfedilmesini içeren tümevarımsal analiz gerçekleştirilmiştir. Bu doğrultuda, çalışmada karşılaşılan hatalar alanyazında yer alan hatalar da göz önünde bulundurularak kategorilere ayrılmış ve adlandırmalar oluşturulmuştur.

Bulgular

Araştırmada elde edilen verilere göre, Denk Kesirler Bilgi Testinden alınabilecek toplam puan 6 iken 5. sınıf düzeyindeki 435 öğrencinin testten aldığı toplam puanların aritmetik ortalaması 2,40 olarak bulunmuştur. Böylece, öğrencilerin denk kesirler bilgisini ölçen testin genelinde düşük bir performans sergilediği sonucuna ulaşılmıştır.

Testteki her bir soruyu doğru ve yanlış cevaplayan öğrencilerin frekans ve yüzde değerleri incelendiğinde ise en yüksek başarının %49,2'lik doğru cevaplanma oranıyla kesrin sembolik gösterimi ile dairesel alan modeli gösterimi arasında geçiş yapabilmeyi gerektiren ikinci soruya ait olduğu görülmektedir. En düşük başarının ise öğrencilerin yalnızca %26,9'unun doğru cevaplayabildiği ve kesrin sembolik gösterimi ile küme modeli gösterimi arasında geçiş yapabilmelerini gözlemek amacıyla bir bağlam içerisinde hazırlanan altıncı soruya ait olduğu anlaşılmaktadır.

Araştırmada soruları yanlış cevaplayan öğrencilerin hatalı yaklaşımları irdelenmiş ve denk kesirler konusunda karşılaşılan hatalar 9 kategoride toplanmıştır. Bu hatalar şu şekilde özetlenebilir:

- Genişletme ve sadeleştirme işlemlerinin çarpma ve bölme algoritmaları ile karıştırılması,
- Alan modelinde eş bütünler oluşturulamaması ya da bir bütünün eş parçalara ayrılamaması,
- Denk kesir elde edilmesi gereken durumlarda genişletme ya da sadeleştirme işlemlerine başvurmaksızın doğrudan kesrin payında yer alan sayıya odaklanılması,
- Tam, yarım ve çeyrek kesir kavramlarının yeterince içselleştirilememesi,
- Denk kesir oluşturmada kesirlerin payları ve paydaları arasında toplamsal ilişki kurulması,
- Denk kesir oluşturma sürecinde birimler arasındaki geçişin doğru bir şekilde sağlanamaması,
- Denk kesirleri karşılaştırmada doğal sayılardaki karşılaştırma gibi düşünülmesi ve bu yüzden kesirler birbirine denk olmasına rağmen birinin diğerinden daha küçük ya da daha büyük olduğunun düşünülmesi,
- Sorunun bağlamından bağımsız şekilde bir kesrin sıfırdan farklı herhangi bir sayıyla doğrudan genişletilmesi ya da sadeleştirilmesi,
- Küme modelinde yeni bir birimin eş büyüklüklerde oluşturulamaması.

Araştırmada gözlenen bu hatalardan en yaygın olanı ise öğrencilerin bir kesri 2 ile çarpmayı ve 2 ile genişletmeyi aynı algoritma olarak görürken benzer şekilde bir kesri 2 'ye bölmeyi ve 2 ile sadeleştirmeyi aynı algoritma olarak düşünmesi olmuştur.

Tartışma ve Sonuç

Öğrencilerin bu araştırmada, iki kesrin birbirine denk olduğunu açıklamadaki alan modeli tercihlerinde bir eğilim dikkat çekmiş ve dikdörtgensel alan modeli kullanmayı tercih eden öğrenciler arasında doğru cevaba ulaşma oranı daha yüksek iken dairesel alan modelini kullanmayı tercih eden öğrenciler arasında ise yanlış cevaplama oranının daha yüksek olduğu görülmüştür. Alan modelleri arasındaki bu performans farkının nedeni ise öğrencilerin dairesel alan modelini eşit parçalara bölmede dikdörtgensel alan modeline göre daha fazla zorluk çekmeleri olabilir. Bu doğrultuda, öğrencilerin dikdörtgensel alan modeli gibi diğer alan modellerine hâkim olduktan sonra dairesel alan modeli ile çalışmaları ve dairesel alan modelinin eş parçalara ayrılmasında çift sayıda eş parçalara ayırma işleminin ardından tek sayıdaki parçalamalara geçilmesi önerilmektedir (Eroğlu vd., 2019).

Araştırmada elde edilen bulgular değerlendirildiğinde öğrencilerin ilk dört soruya nazaran beşinci ve altıncı soruları anlamlandırmada zorlandıkları ve bu sorularda kendilerini daha zor ifade ettikleri görülmüştür. Kesirlerin öğretiminde alan, uzunluk, küme modelleri gibi çeşitli modellere yer verilmesi oldukça kullanışlı ve faydalı olmakla birlikte (Hansen vd., 2016; Van de Walle vd., 2013) öğrencilerin alan modeline daha çok aşina oldukları ve diğer modellerde daha fazla zorlandıkları aşikâr. Ayrıca, öğrencilerin beşinci ve altıncı soruyu anlamlandırmada güçlük çekmelerinin bir diğer nedeni de bu soruların bir bağlam içerisinde verilmesi olabilir.

Son olarak araştırmanın bulguları doğrultusunda, denk kesirlerin elde edilmesi sürecindeki birim kesir kavramının önemine dikkat çekmek gerekmektedir. Çünkü birim kesir oluşturabilme denk kesirleri anlamanın temelini oluşturmaktadır (Lamon, 2012). Bu çalışmada özellikle birden fazla algoritma kullanımını içeren dördüncü soruda, öğrencilerin matematiksel işlemleri uygulamadan önce birimler arasındaki farklılığı göz ardı ettikleri ve bunun sonucunda farklı birimler arasında doğru bir geçiş yapamadıkları görülmüştür.

Öneriler

Yapılan incelemeler ve elde edilen bulgular neticesinde öğrencilerin kesirlerin sembolik gösterimini kullanmada, dikdörtgensel ve dairesel alan modeli gösterimlerinde, tipik ve simetrik örneklerin yer aldığı durumlarda daha başarılı oldukları görülmüştür. Öğrencilerin bu başarılı performansları edindikleri deneyimler ile orantılı olabileceğinden uzunluk modeli, küme modeli, tipik olmayan örnekler, asimetrik örnekler ve örnek olmayan durumlar ile daha fazla yaşantı geçirmelerine fırsat tanınması önerilmektedir.

Etik Kurul İzin Bilgileri

Araştırmanın etik kurul izni, Hacettepe Üniversitesi Etik Komisyonu tarafından 25.01.2022 tarihine yapılan toplantıda alınmıştır.

Araştırmanın Etik Taahhüt Metni

Yapılan bu çalışmada bilimsel, etik ve alıntı kurallarına uyulduğu; toplanan veriler üzerinde herhangi bir tahrifatın yapılmadığı, karşılaşılacak tüm etik ihlallerde "Cumhuriyet Uluslararası Eğitim Dergisi ve Editörünün" hiçbir sorumluluğunun olmadığı, tüm sorumluluğun Sorumlu Yazara ait olduğu ve bu çalışmanın herhangi başka bir akademik yayın ortamına değerlendirme için gönderilmemiş olduğu sorumlu yazar tarafından taahhüt edilmiştir.

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Appendix 1. Equivalent Fractions Knowledge Test

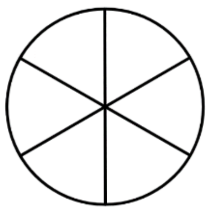
1)

$$\frac{2}{6} = \frac{1}{3}$$

How would you explain that the fractions given above are equivalent? Explain in two different ways.

Explanation:

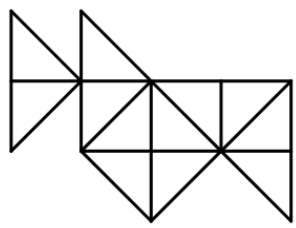
2)



Please shade $\frac{3}{3}$ of the model divided into equal parts above and explain how you think.

Explanation:

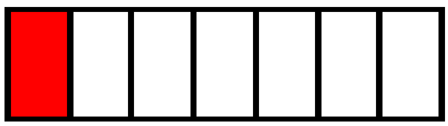
3)



Please shade $\frac{4}{6}$ of the model consisting of equal parts above and explain how you think.

Explanation:

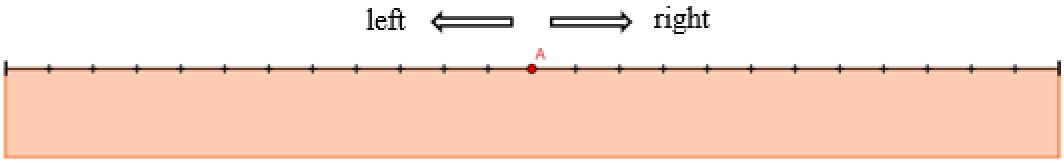
4)



How many more parts must be shaded to make $\frac{6}{14}$ of the above model look shaded? Explain how you solved it.

Explanation:

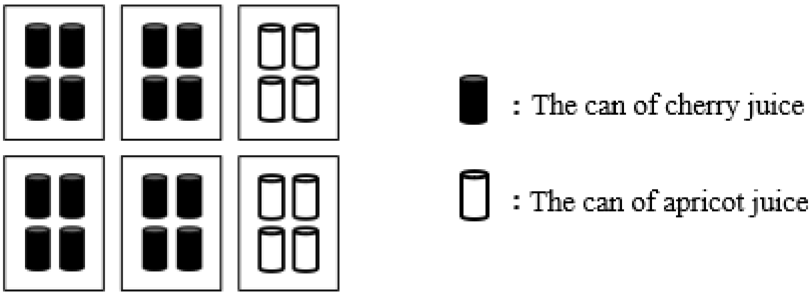
5)



There is point A in the middle of the ruler. Starting from point A towards the left of the ruler, point B is placed at a distance of $\frac{5}{6}$ the length of the left part. Similarly, starting from point A towards the right of the ruler, point C is placed at a distance of $\frac{20}{24}$ the length of the right part. According to the given situation, compare the distances of points B and C from the point A, and explain how you think.

Explanation:

6)



A grocery store divided the cherry and apricot juices into 6 boxes, as above, with four juices of the same type in each box. This grocery store has enough boxes, and cans of juices are wanted to rearrange so that each box contains the same type of juices but different amount than four. Accordingly, how many of the boxes containing apricot juices are in all boxes? Explain in two different arrangements.

Explanation: