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Investigation of Middle School Students' Probabilistic Reasoning Levels in Terms of Some Variables#

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Research Article

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ABSTRACT

The purpose of the present study was to identify the probabilistic reasoning levels of sixth, seventh and eighth grade students, and the study also aimed to investigate the relationships between level of probabilistic reasoning and gender, grade level and mathematics achievement. The study employed the descriptive survey and relational survey models, and the study group was comprised of 286 students. To identify the probabilistic reasoning levels of students, the probabilistic reasoning scale developed by the researchers of the present study was utilized. Students' probabilistic reasoning was examined for six key concepts called sample space, experimental probability of an event, theoretical probability of an event, probability comparisons, conditional probability and independence. Descriptive statistics were used to identify students' levels of probabilistic reasoning, and Chi-square analyses were conducted to reveal the relationships between reasoning levels and gender, grade level and mathematics achievement. The analyses revealed that most of the students' reasoning skills were at level 3 in the concepts of the theoretical probability of an event and probability comparisons and at level 1 in the other concepts. A positive relationship was revealed between gender and the concept of sample space, between grade level and all the other concepts, and between mathematics achievement and the concepts of sample space, theoretical probability of an event, probability comparisons and conditional probability.

Keywords: Middle school student, probability, probabilistic reasoning, cross-age

Ortaokul Öğrencilerinin Olasılıksal Akıl Yürütme Beceri Düzeylerinin Bazı Değişkenler Açısından İncelenmesi

Bilgi
#Bu çalışma yüksek lisans tezinin bir parçasıdır.
**Sorumlu yazar*

Süreç
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Bu çalışma ön inceleme sürecinde ve yayımlanmadan önce iThenticate yazılımı ile taranmıştır.

ÖZ

Bu çalışmada; ortaokul altıncı, yedinci ve sekizinci sınıf öğrencilerinin olasılıksal akıl yürütme düzeylerinin belirlenmesi ve bu düzeylerin cinsiyet, sınıf düzeyi ve matematik başarısı ile ilişkisinin incelenmesi amaçlanmıştır. Araştırma 286 öğrenci ile gerçekleştirilmiş olup, betimsel ve ilişkisel tarama modelleri kullanılmıştır. Öğrencilerin olasılıksal akıl yürütme düzeylerinin belirlenmesi için araştırmacılar tarafından geliştirilen "olasılıksal akıl yürütme ölçeği" kullanılmıştır. Olasılıksal akıl yürütme örnek uzay, bir olayın deneysel olasılığı, bir olayın teorik olasılığı, olasılık karşılaştırmaları, bağımlı olasılık ve bağımsızlık olarak adlandırılan altı anahtar kavram için ele alınmıştır. Öğrencilerin olasılıksal akıl yürütme düzeylerinin belirlenmesinde betimsel istatistikler kullanılmıştır. Ki-kare analizle öğrencilerin bu düzeyleri ile cinsiyet, sınıf düzeyi ve matematik başarısı arasındaki ilişki belirlenmeye çalışılmıştır. Analizler sonucunda, bir olayın teorik olasılığı ve olasılık karşılaştırmaları alt kavramlarında öğrencilerin çoğunluğunun üçüncü düzey, diğer alt kavramlarda ise birinci düzey akıl yürütme becerisine sahip oldukları tespit edilmiştir. Cinsiyet değişkeni ile sadece örnek uzay arasında ilişkiye ulaşılmıştır. Sınıf düzeyi ile tüm kavramlar arasında istatistiksel olarak anlamlı düzeyde pozitif bir ilişki bulunmuştur. Matematik başarısı ile bağımsızlık ve bir olayın deneysel olasılığı dışındaki tüm kavramlar arasında pozitif bir ilişki olduğu belirlenmiştir.

Anahtar Kelimeler: Ortaokul öğrencisi, olasılık, olasılıksal akıl yürütme, yaş karşılaştırmalı çalışma

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Introduction

Probability is the one of the branches in mathematics that deals with the frequency of occurrence of an event (Altun, 2010). It has an important place in mathematics and is closely related to other branches of mathematics, particularly to the branches of numbers and geometry (NCTM, 2000).

With the understanding of the importance of probability in daily life and in various business areas, probability became part of the mathematics curriculum in many countries towards the end of the 19th century (Gürbüz, 2010; Kazak, 2010a). The National Council of Teachers of Mathematics [NCTM] (2000) emphasized that probability teaching should start from an early age and stated that preschool children encountered the concept of probability informally in daily with statements starting with such expressions as “maybe”. The aim of teaching probability is to enable students to make strong predictions about the probability of the occurrence of an event (Altun, 2010). From an early age, our intuition plays a role in this decision-making and estimation process (Kazak, 2010b). In order to develop the accuracy of these and to develop and promote the use of scientific reasoning decision-making and estimation processes, the subject of probability started to be included in mathematics education. Its introduction into the curriculum in Turkey took place in the 1960s. In Turkey, the topic of probability was addressed only in the curricula of grades 8 and 9 before the year 2000 (Bulut, 2001).

Altun (2010) defines reasoning as “a way of thinking that will enable people to understand what is happening around them, to see the relationship between the causes and effects of events and to benefit from them” (p. 7). Reasoning is a skill that has an important place in every field of mathematics. Its importance in the field is indicated in NCTM resources stating that mathematics itself is reasoning (NCTM, 1989). Developing students’ reasoning skills is among the goals of mathematics education (Fitzgerald, 1996).

In order to understand mathematics, reasoning is necessary, and correspondingly reasoning is a basic requirement to understand probability, which is a branch of mathematics. Probabilistic reasoning refers to the ability to understand and explain probabilistic processes. Probabilistic reasoning involves the ability of making models similar to random events, identifying appropriate data to predict probabilities, using related situations when solving a problem and thinking about the situations in which subjective probabilities can be used (Jolliffe, 2005). Basic categories of probabilistic reasoning are defined to involve the following: the ability to distinguish between randomness and causation, the ability to balance the psychological and formal elements of probability, and the ability to understand that the criteria for reflecting on a random situation are different from those that will be applied in the selection of a decision (Borovcnik & Kapadia, 2018).

The first study on probabilistic reasoning was conducted by Piaget and Inhelder (1975). This study, which is considered to be a seminal and basic psychological study on the development of probabilistic reasoning in children (Way, 2003), explained the development of probability concepts in children with age (Drier, 2000). Students use different types of reasoning depending on their subjective characteristics, and their social experiences and intuitions affect their thoughts and decisions (Fischbein, 1975; Fischbein & Schnarch, 1997; Sharma, 2005; Rubel, 2007, 2009; Williams & Amir, 1995).

There is a relationship between students’ mathematical reasoning and probabilistic reasoning, (Gürbüz & Erdem, 2014). Sezgin Memnun (2008) states that a student with underdeveloped mathematical reasoning will have difficulty in learning the subjects of probability. He adds that skill develops with age, and that the teacher, the student’s attitude and the education system are effective in the development of the reasoning skill. Thus, there seems to be agreement in the literature that reasoning has an important place in learning the subject of probability.

Jones et al. (1997) developed a theoretical framework that systematically describes and characterizes four key concepts of children’s probabilistic reasoning: probability of an event, sample space, conditional probability and probability comparisons (Jones et al., 1997). Four levels were determined for these concepts. A rubric was developed to systematically describe the features that can be observed at each of the four levels of these concepts (Jones et al., 1997). In another study by Tarr and Jones (1997), the concept of independence was considered as another key concept in reasoning; hence, the rubric was expanded. The same researchers defined the probability of an event as two separate concepts: theoretical probability of an event and experimental probability of an event, and created the final version of the rubric based on these concepts. Of these concepts for probabilistic reasoning, the most fundamental one was reported to be sample space. In this concept, the students are expected to list the outputs of one or more experiments. The next concept is the experimental probability of an event, which refers to the determination of the frequency of an event based on experimentation or simulation. The third concept, theoretical probability of an event, is the determination of sample space by analyzing it using number, symmetry and simple geometry measurements. The relevant literature has observed that the experimental probability result obtained with the increase in the number of experiments approximates the theoretical probability result of the same event. The experimental probability of an event and the theoretical probability of an event are related concepts. However, the literature reports that primary and middle school students cannot see this relationship clearly. The fourth concept, probability comparisons, is used to determine which of the two probability situations is more likely to come up with a target event or whether they have an equal chance to occur for the target event. The fifth concept, the conditional probability, is the change of the probability of the event that we want to happen with the occurrence of another event. The sixth concept is independence. Here, the occurrence of an event and the probability of another event that we want to happen do not affect each other; that is, the probabilities of their occurrences are independent of each other. For experimental probability an example question is given as, “Miss Pierce did 20 practice draws before she did the draw to decide the president. Her results were as follows: Jennifer, 3 times; Martina, 3; Monica, 4; Philip, 2; and Sergio, 8. On the basis of these results, who has the best chance for president, or is it not possible to say? Explain your thinking. Suppose Miss Pierce did 100 practice draws; who do you think the result would be? Give a number for each student and justify your thinking” (Jones et al., 1999, p. 148).

In the first level of the rubric developed by Jones et al. (1999), students consider probability situations from a limited perspective. They tend to focus subjectively rather than scientifically on what can happen. Hence, they use a subjective point of view rather than quantitative reasoning. Students at the second level are in transition between subjective and informal quantitative reasoning. Despite fully describing the outcomes of an experiment, they create a weak link between sample space and probability and often revert to subjective reasoning. Those who reason at this level in conditional probability do not recognize probabilistic situations where the probability changes as the sample space is reduced. Students at the third level use more systematic strategies when listing the outputs of one or more experiments. The most substantial change in the thinking of those types of reasoning at this level is the tendency to use more quantitative reasoning when determining probabilities and conditional probabilities. Students make more use of such comparison expressions such *as more likely, less likely*, or *equally likely*, rather than the classic probability expressions, and sometimes turn to representations such as *3 out of 5*. Students demonstrating reasoning at the fourth level use systematic reasoning to determine the outcomes of an experiment and to determine their quantitative probabilities in both experimental and theoretical situations.

Probabilistic reasoning has a special place in mathematical reasoning (Jones et al., 1999). The history of probabilistic reasoning is considered to go back to the 17th century when it is believed to have been used in daily life; however, it has been a part of school curricula only in the last 50 years (Koyuncu, 2017). Similarly, probabilistic thinking entered the school curricula in the Turkish education system only in recent years. Moreover, probability has only recently been considered a sub-branch of mathematics. Therefore, the subject of probability is a relatively new subject in mathematics education compared to other subjects. Hence, there is not as much comprehensive research on probability as there is on other subjects.

As in other countries, there are some problems in the teaching of probability in Turkey. Gürbüz (2006) identified the following reasons underlying the difficulties students experience in learning the subject of probability: most students have undeveloped reasoning skills and they a) memorize the rules and formulas, b) make inaccurate comments by making subjective judgments with the information they have obtained from daily life and c) produce solutions by themselves and have negative attitudes towards the subject. “Probability” is generally connotated with games of chance; thus, it is seen as an area where trial and error is resorted to, and which contains prejudices such as luck. Since probabilistic reasoning is a- relatively new field, there have not been significant changes in the way it is perceived. Thus, there are some obstacles in the teaching and learning of probabilistic concepts. Scientific thinking should be resorted to rather than prejudices such as luck, intuitiveness, trial and error in probabilistic reasoning. Hence, the teaching of probability to students should initially be focused on eliminating these prejudices discouraging the use of the trial and error method, and most importantly, raising awareness in the benefits of the reasoning skill.

Hence, by addressing these gaps in the related literature, the current study will contribute to the literature on probabilistic reasoning and the scale developed within the scope of the study will be useful in teaching probability for educators and teachers. More specifically, the current study aimed to identify the levels of probabilistic reasoning skill of 6th, 7th and 8th grade middle school students and investigate the relationship between their reasoning skill and gender, grade level and level of achievement in mathematics .

The problems and sub-problems of the current study were as follows:

1- What are the probabilistic reasoning levels of *6th, 7th and 8th* grade middle school students?

2- Is there a significant correlation between the probabilistic reasoning levels of *6th, 7th and 8th* grade middle school students and their gender, grade level and mathematics achievement?

a. Is there a significant correlation between the probabilistic reasoning levels of *6th, 7th and 8th* grade middle school students and their gender?

b. Is there a significant correlation between the probabilistic reasoning levels of *6th, 7th and 8th* grade students and their grade level?

c. Is there a significant correlation between the probabilistic reasoning levels of 6th, 7th and 8th grade students and their mathematics achievement?

Methodology

Research Model

In the current study, a descriptive survey model was employed to identify the probabilistic reasoning levels of 6th-to-8th grade middle school students. Moreover, a correlational research design was used to investigate whether there was a significant correlation between the students' levels of probabilistic reasoning and their gender, grade level and mathematics achievement.

Study Group

The random sampling method was used to select three middle schools in the province of Yalova to participate in the study. The students attending these schools were from medium level socio-economic families. The study group of the current study was comprised of 286 participants, who were randomly selected middle school students from grades 6, 7, and 8 from among the classes of these three schools. In the 2014 academic year, when the study was conducted, the 2013 mathematics curriculum was in effect. However, during the data collection stage of the current study, the 2009 curriculum was implemented in all the classes as it was a transitional period . Since 5th graders are taught according to the new program, they are not included in the study. In the 2009 curriculum, the subjects on probability addressed across different grade levels of middle school were as follows: types of events at the 6th grade level; discrete and non-discrete events at the 7th grade level; permutation, conditional and non-conditional probability and combination at the 8th grade level. In the 2013 renewed curriculum, the learning outcomes related to probability at the 6th and 7th grade levels were removed and were only included at the 8th grade level.

Data Collection Tools

In the current study, a new measurement tool was created based on the rubric and theoretical structure developed by Jones et al. (1999). They define probabilistic reasoning in four hierarchically progressing levels under the concepts of sample space, experimental probability of an event, theoretical probability of an event, probability comparisons, conditional probability and independence. Students at the first level are in transition between intuitive and subjective reasoning, and students at the second level are in transition between subjective and informal quantitative reasoning. Students at the third level exhibit informal quantitative reasoning, while students at the fourth level demonstrate quantitative reasoning. The Probabilistic Reasoning Scale developed for the current study was also constructed based on this framework. The validity of the Scale was established through expert review. Opinions of three experts were taken for the measurement tool consisting of 25 draft items. Corrections were made on the basis of the expert review showing that the questions were suitable for their purpose, but that there were items that could be difficult to understand. In the visuals of the rotation questions, the tip of the arrow was clarified and the confusion of the place where the arrow stopped was eliminated. A pilot study was conducted on 54 students from 6th, 7th and 8th grades in a state middle school located in Yalova. Expert opinion was sought again for these 15 draft items.

Necessary corrections were made in line with the opinions of three different experts. A second pilot study was conducted on 102 students studying in 6th, 7th and 8th grades in another state middle school located in the central district of Yalova province with these 15 draft items. Since there was no problem experienced in the pilot study, the expert opinions received for this measurement tool consisting of open-ended questions were found to be sufficient for the main application. Thus, the measuring tool was given its final form with three items in each of the following concepts: sample space, probability comparisons, conditional probability and independence. There is one item for the concept of experimental probability of an event and two items for the concept of theoretical probability of an event.

Since the aim of the study was to investigate students' probabilistic reasoning, the items in the measurement tool were formed as open-ended questions, which were created by drawing on the relevant literature. Since four levels were determined for each sub-concept in the measurement tool, the scoring of the items was made between 1 and 4. A detailed rubric was prepared to examine student answers to the items in the measurement tool. Examination of the rubric in relation to the student answers to each item in the sample space dimension in the measurement tool is given in Table 1.

Data Analysis

Descriptive statistics method was used to understand the levels of the probabilistic reasoning of middle school students. To identify the probabilistic reasoning levels, four levels in the rubric were taken as the basis. Subsequently, the sub-concepts were evaluated within themselves. This coding process was applied twice by the researcher. There was a three-week time interval between the completion of the first coding and the second coding. Third opinion was taken in cases where different evaluations emerged. The Statistical Program Package for Social Sciences (SPSS) was used to conduct the analyses in order to investigate whether there was a relationship between the probabilistic reasoning levels of these students and the variables of gender, grade level and mathematics achievement. Chi-square analysis was run to examine whether there was a relationship between students' probabilistic reasoning levels and other variables. The level of significance for all the analyses was set chosen as $p<0.05$.

The Kramer V coefficient analysis and the Kendall Tau B coefficient analysis were performed to interpret the size of the relationship in cases where there was a statistical relationship.

Table 1.Examination of the probabilistic reasoning levels rubric for sample items and answers

Sample Space/Levels	Level 1	Level 2	Level 3	Level 4
At a pizza restaurant, you can have your own pizza made with the ingredients you choose. You can choose from among four different ingredients: olive, sausage, mushroom and salami. Reyhan wants to order a pizza with two different ingredients. Reyhan can choose her pizza from how many different options? Why? (PISA, 2000)	Lists an incomplete set of outputs for a one-stage experiment. Possible Answers: *"4 different because she has already written the ingredients she can choose." "She can choose from among 4 x 2 = 8 different options." "I would choose sausage and salami because I like them."*	Can list the complete set of outputs for a one-stage experiment and sometimes for a two-stage experiment. Possible Answers: *"She can choose 6 because there is no other ingredient." "She can choose 6 different ingredients because with each ingredient, another ingredient is added."*	Consistently lists the results of a two-stage experiment using a partially generative strategy. Possible Answers: *"12 different options; Olive-sausage, olive – mushroom, olive-salami, salami-olive, salami-mushroom, salami-sausage, mushroom-sausage, mushroom-salami mushroom-olive, sausage-mushroom, sausage-salami, sausage-olive"*	Adopts and implements a generative strategy to provide a complete list of outputs for two- and three-stage experiments. Possible Answers: *"olive-sausage olive-mushroom olive-salami sausage-mushroom sausage-salami mushroom-salami 6 options"*

Table 2.Descriptive Statistics for Probabilistic Reasoning Levels

Concepts		Level 1	Level 2	Level 3	Level 4	Total
Sample Space	f	213	73	-	-	286
	%	74.4	25.6	-	-	100
Experimental Probability	f	176	38	28	44	286
	%	61.5	13.2	9.8	15.5	100
Theoretical Probability	f	2	22	239	23	286
	%	0.7	7.7	83.5	8,1	100
Probability Comparison	f	38	6	223	19	286
	%	13.3	2.1	78	6.6	100
Conditional Probability	f	169	109	4	4	286
	%	59.1	38.1	1.4	1.4	100
Independence	f	211	33	28	14	286
	%	73.8	11.6	9.8	4.8	100

Ethical procedures. The ethical permissions of the research were discussed and approved by the ethics committee of Hacettepe University on 14th May 2015 with the number 435-1442.

Findings

1. Findings and Interpretations related to the 1st Research Question "What are the levels of probabilistic reasoning of sixth, seventh and eighth grade middle school students?"

Within the context of this research question, the probabilistic reasoning levels of the students were determined separately for the six sub-concepts. To this end, descriptive analysis was conducted. The results of the descriptive analysis revealing the students' levels of probabilistic reasoning are presented in Table 2 below.

As can be observed in Table 1, most of the students were at level 1 in terms of probabilistic reasoning in sample space, in experimental probability of an event, in conditional probability and in independence. On the other hand, most of the students were at level 3 in theoretical probability of an event, in probability comparisons, When the curriculum in effect during the academic year in which the scale was administered was examined, it was observed that the learning outcomes of calculating the probability of an event and making probability comparisons were included in the curriculum.

Thus, the fact that the students frequently encountered question types that included the concepts of theoretical probability of an event and probability comparisons was believed to be the reason underlying the levels of these concepts being found to be higher than those of the other concepts.

However, even though the learning outcomes related to the concepts of sample space, experimental probability of an event and independence were also included in the same curriculum, the levels of probabilistic reasoning for these concepts were found to be low.

The reason for this may be that these learning outcomes were not included in the question types, that the students had misconceptions, that the students could not fully understand the question and that they had prejudices.

2. Findings and Interpretations related to the Sub-Question "Is there a significant correlation between the levels of probabilistic reasoning of 6th, 7th and 8th grade middle school students and their gender?"

This sub-question of the study sought to investigate whether there was a correlation between the probabilistic reasoning levels of the middle school students and their gender.

To this end, the existence of a correlation between six sub-concepts of probabilistic reasoning and gender was tested by performing a chi-square analysis, and in cases where there was a significant correlation the size of the correlation was interpreted by applying the Kramer V test. The obtained findings are summarized in Table 3.

As can be observed in Table 5, there is a significant correlation between gender and the levels of probabilistic reasoning possessed by the students for the concept of sample space (X^2=9.69, df=3, p<0.05). The Kramer V value was calculated for the direction and strength of this correlation. The Kramer V value was found to be 0.18. According to this value, it can be said that there is a low correlation between gender and the levels of probabilistic reasoning possessed for the concept of sample space (Özbay, 2008).

On the other hand, no significant correlation was found between the levels of probabilistic reasoning possessed by the students for the concept of experimental probability of an event and gender (X^2=4.89, df=3, p=0.180). Similarly, no significant correlation was found between the levels of probabilistic reasoning possessed by the students for the concept of theoretical probability of an event and gender (X^2=2.99, df=3, p= 0.392). Nor was there a significant correlation between the levels of probabilistic reasoning possessed by the students for the concept of probability comparisons and gender (X^2=0.33, df=3, p=0.953). Moreover, no significant correlation was found for the concepts of conditional probability (X^2=0.36, df=3, p=0.948) and independence (X^2=1.83, df=3, p=0.608).

3. Findings and Interpretations related to the Sub-Question "Is there a significant correlation between the levels of probabilistic reasoning of 6th, 7th and 8th grade middle school students and their grade level?"

The aim of this sub-question was to investigate whether there was a significant correlation between the probabilistic reasoning levels of the 6th, 7th and 8th grade middle school students and the grade level variable. To this end, the existence of a correlation between six sub-concepts of probabilistic reasoning and grade level was tested by performing a chi-square analysis, and in cases where there was a significant correlation, the size of the correlation was interpreted by applying the Kendall Tau B test. The obtained findings are summarized in Table 4.

Table 3. The correlations between the probabilistic reasoning levels of the middle school 6th, 7th and 8th grade students and their gender, and the results of chi-square analysis

Concepts	Gender	Levels 1		2		3		4		Total		X^2	df	p
		f	%	f	%	f	%	f	%	f	%			
Sample Space	Female	101	66.9	50	33.1	0	0	0	0	151	100	9.69	3	*0.002
	Male	112	83	23	17	0	0	0	0	135	100			
	Total	213	74.5	73	25,5	0	0	0	0	286	100			
Experimental Probability	Female	94	62.3	15	9.9	14	9.3	28	18.5	151	100	4.89	3	0.180
	Male	82	60.7	23	17	14	10.4	16	11.9	135	100			
	Total	176	61.5	38	13.3	28	9.8	44	15.4	286	100			
Theoretical Probability	Female	2	1.3	14	9.3	123	81.5	12	7.9	151	100	2.99	3	0.392
	Male	0	0	8	5.9	116	85.9	11	8.1	135	100			
	Total	2	0.7	22	7.7	239	83.6	23	8	286	100			
Probability Comparison	Female	21	13.9	3	2	118	78.1	9	6	151	100	0.33	3	0.953
	Male	17	12.6	3	2.2	105	77.8	10	7.4	135	100			
	Total	38	13.3	6	2.1	223	78	19	6,6	286	100			
Conditional Probability	Female	87	57.6	60	39.7	2	1.3	2	1.3	151	100	0.36	3	0.948
	Male	82	60,7	49	36.3	2	1.5	2	1,5	135	100			
	Total	169	59.1	109	38.1	4	1.4	4	1.4	286	100			
Independence	Female	108	71.5	17	11.3	18	11.9	8	5.3	151	100	1.83	3	0.608
	Male	103	76.3	16	11.9	10	7.4	6	4.4	135	100			
	Total	211	73.8	33	11.5	28	9.8	14	4.9	286	100			

$*p<0.05$

Table 4. The Correlations between the Probabilistic Reasoning Levels of Students and their Grade Level and the Results of Chi-square Analysis

Concepts	Grade Level	Levels 1 f	1 %	2 f	2 %	3 f	3 %	4 f	4 %	Total f	Total %	X^2	df	p
Sample Space	6	63	80.8	15	19.2	0	0	0	0	78	100	9.12	6	*0.010
	7	76	80.9	18	19.1	0	0	0	0	94	100			
	8	74	64.9	40	35.1	0	0	0	0	114	100			
	Total	213	74.5	73	25.5	0	0	0	0	286	100			
Experimental Probability	6	57	73.1	6	7.7	9	11.5	6	7.7	78	100	25.02	6	*0.000
	7	66	70.2	14	14.9	5	5.3	9	9.6	94	100			
	8	53	46.5	18	15.8	14	12.3	29	25.4	114	100			
	Total	176	61.5	38	13.3	28	9.8	44	15.4	286	100			
Theoretical Probability	6	0	0	1	1.3	73	93.6	4	5.1	78	100	18.03	6	*0.006
	7	2	2.1	14	14.9	70	74.5	8	8.5	94	100			
	8	0	0	7	6.1	96	84.2	11	9.6	114	100			
	Total	2	0,7	22	7.7	239	83.6	23	8	286	100			
Probability Comparison	6	17	21,8	1	1.3	55	70.5	5	6,4	78	100	13.86	6	*0.031
	7	15	16	3	3.2	72	76.6	4	4,3	94	100			
	8	6	5.3	2	1.8	96	84.2	10	8.8	114	100			
	Total	38	13.3	6	2,1	223	78	19	6.6	286	100			
Conditional Probability	6	48	61.5	30	38.5	0	0	0	0	78	100	19.41	6	*0.004
	7	66	70.2	28	29.8	0	0	0	0	94	100			
	8	55	48.2	51	44.7	4	3.5	4	3.5	114	100			
	Total	169	59.1	109	38.1	4	1.4	4	1.4	286	100			
Independence	6	64	82.1	10	12.8	1	1.3	3	3.8	78	100	39.38	6	*0.000
	7	83	88.3	6	6.4	4	4.3	1	1.1	94	100			
	8	64	56.1	17	14.9	23	20.2	10	8.8	114	100			
	Total	211	73.8	33	11.5	28	9.8	14	4.9	286	100			

***p*<0.05**

As can be observed in Table 4, there is a significant correlation between grade level and the levels of probabilistic reasoning possessed by the students for the concept of sample space (X^2=9.12, df=6, p<0.05). Similarly, there is a significant correlation between grade level and the levels of probabilistic reasoning possessed by the students for the concept of experimental probability of an event (X^2=25.02, df=6 p<0.05). Moreover, a significant correlation was also found between grade level and the levels of probabilistic reasoning possessed by the students for the concepts of theoretical probability of an event (X^2=18.03, df=6, p<0.05) and probability comparisons (X^2=13.86, df=6, p<0.05). A significant correlation was also found between grade level and the levels of probabilistic reasoning possessed by the students for the concepts of conditional probability (X^2=19.41, df=6, p<0.05) and independence (X^2=39.38, df=6, p<0.05).

In order to interpret these correlations, Kendall Tau B coefficient was used. The Kendall Tau B values calculated for the correlations between grade level and the levels of probabilistic reasoning possessed by the students for the concepts of sample space, probability comparisons and conditional probability were found to be 0.15, 0.16 and 0.14, respectively. These values show low correlations. On the other hand, moderate level correlations were found between grade level and the levels of probabilistic reasoning possessed by the students for the concepts of experimental probability of an event (Kendall Tau B=0.22) and independence (Kendall Tau B=0.25).

A very low correlation was found between grade level and the levels of probabilistic reasoning possessed by the students for the concept of theoretical probability of an event (Kendall Tau B=0.01).

4. Findings and Interpretations related to the Sub-Question "Is there a significant correlation between the levels of probabilistic reasoning of 6th, 7th and 8th grade middle school students and their mathematics achievement?"

In regards to this sub-question, the aim was to investigate the correlation between the probabilistic reasoning levels of the students and their mathematics achievement.

To this end, the existence of a correlation between six sub-concepts of probabilistic reasoning and mathematics achievement was tested by performing a chi-square analysis, and in cases where there was a significant correlation, the size of the correlation was interpreted by applying the Kendall Tau B test. The obtained findings are presented in Table 5.

As can be observed in Table 5, there is a significant correlation between mathematics achievement and the levels of probabilistic reasoning possessed by the students for the concept of sample space (X^2=13.62, df=12, p<0.05).

Table 5. The correlations between the probabilistic reasoning levels of the students and their mathematics achievement and the results of chi-square analysis

Concepts	Math Achievement	Levels 1		2		3		4		Total		X^2	df	p
		f	%	f	%	f	%	f	%	f	%			
Sample Space	1	37	92.5	3	7.5	0	0	0	0	40	100	13.62	12	*0.009
	2	37	71.2	15	28.8	0	0	0	0	52	100			
	3	54	80.6	13	19.4	0	0	0	0	67	100			
	4	41	71.9	16	28.1	0	0	0	0	57	100			
	5	44	62.9	26	37.1	0	0	0	0	70	100			
	Total	213	74.5	73	25.5	0	0	0	0	286	100			
Experimental Probability	1	28	70	4	10	2	5	6	15	40	100	9.67	12	0.644
	2	33	63.5	8	15.4	6	11.5	5	9.6	52	100			
	3	41	61.2	6	9	5	7.5	15	22.4	67	100			
	4	36	63.2	8	14	7	12.3	6	10.5	57	100			
	5	38	54.3	12	17.1	8	11.4	12	17.1	70	100			
	Total	176	61.5	38	13.3	28	9.8	44	15.4	286	100			
Theoretical Probability	1	0	0	8	20	31	77.5	1	2.5	40	100	30,10	12	*0,003
	2	2	3.8	6	11.5	41	78.8	3	5.8	52	100			
	3	0	0	3	4.5	60	89,6	4	6	67	100			
	4	0	0	4	7	44	77.2	9	15,8	57	100			
	5	0	0	1	1,4	63	90	6	8.6	70	100			
	Total	2	0.7	22	7.7	239	83.6	23	8	286	100			
Probability Comparison	1	15	37.5	2	5	23	57.5	0	0	40	100	41.58	12	*0.000
	2	5	9.6	3	5.8	42	80.8	2	3.8	52	100			
	3	8	11.9	1	1.5	55	82.1	3	4.5	67	100			
	4	4	7	0	0	49	86	4	7	57	100			
	5	6	8.6	0	0	54	77.1	10	14.3	70	100			
	Total	38	13.3	6	2.1	223	78	19	6.6	286	100			
Conditional Probability	1	36	90	4	10	0	0	0	0	40	100	34.58	12	*0.001
	2	38	73.1	13	25	1	1.9	0	0	52	100			
	3	37	55.2	27	40.3	2	3	1	1.5	67	100			
	4	27	47.4	28	49.1	0	0	2	3.5	57	100			
	5	31	44.3	37	52.9	1	1,4	1	1.4	70	100			
	Total	169	59.1	109	38.1	4	1.4	4	1.4	286	100			
Independence	1	34	85	5	12.5	1	2.5	0	0	40	100	11.48	12	0.488
	2	40	76.9	6	11.5	5	9.6	1	1.9	52	100			
	3	46	68.7	8	11.9	9	13.4	4	6	67	100			
	4	43	75.4	6	10.5	6	10.5	2	3.5	57	100			
	5	48	68.6	8	11.4	7	10	7	10	70	100			
	Total	211	73.8	33	11.5	28	9.8	14	4.9	286	100			

$*p<0.05$

On the other hand, no significant correlation was found between mathematics achievement and the levels of probabilistic reasoning possessed by the students for the concept of experimental probability of an event ($X^2=9.67$, df=12, p=0.644). Similarly, no significant correlation was found between mathematics achievement and the levels of probabilistic reasoning possessed by the students for the concept of independence ($X^2=11.48$, df=12, p=0.488).

A significant correlation was found between mathematics achievement and the levels of probabilistic reasoning possessed by the students for the concepts of theoretical probability of an event ($X^2=30.10$, df=12, $p<0.05$), probability comparisons ($X^2=41.58$, df=12, $p<0.05$) and conditional probability ($X^2=34.58$, df=12, $p<0.05$).

In order to interpret the significant correlations, Kendall Tau B coefficient was used. A moderate level of correlation was found between mathematics achievement and the levels of probabilistic reasoning possessed by the students for the concepts of conditional probability (Kendall Tau B=0.26) and probability comparisons (Kendall Tau B= 0.25). A low correlation was found between mathematics achievement and the levels of probabilistic reasoning possessed by the students for the concepts of theoretical probability of an event (Kendall Tau B= 0.18) and sample space (Kendall Tau B=0.15).

Discussion and Conclusion

It is evident that reasoning has an important place in learning the subject of probability. When studies on the difficulties experienced in learning the subject of probability were examined, it was realized that there was a need for examining in detail what students thought about probability, how they reasoned and how they produced solutions when encountered problems. Therefore, in the current study, 6th, 7th and 8th grade middle school students' levels of probabilistic reasoning and whether these students' levels of reasoning were related to gender, grade level and mathematics achievement were investigated. By conducting a descriptive analysis on the results of the probabilistic reasoning test developed in the current study, the probabilistic reasoning levels of the students participating in the study were identified for the six sub-concepts. The majority of students were found to be at level 1 in probabilistic reasoning for the concept of sample space, level 1 for the concept of experimental probability of an event, level 3 for the concept of theoretical probability of an event, level 3 for the concept of probability comparisons, level 1 for the concept of conditional probability and level 1 for the concept of independence. It can be concluded that the students' probabilistic reasoning for the concepts of theoretical probability of an event and probability comparisons are concentrated at level 3, unlike other concepts, and this might stem from the existence of more learning outcomes related to these concepts. Since the curriculum includes learning outcomes related to these concepts, students may frequently be encountering similar questions in their textbooks. Thus, they may have answered the items related to these concepts more easily and more accurately than the items related to the other concepts. It was concluded that because of their experiences of similar questions, the students exhibited higher level of reasoning while answering the items related to these concepts. Although the curriculum includes learning outcomes, such as "Explains terms such as experiment, output, sample space, event, random selection, and equiprobability by relating them to a situation" (MEB, 2009) from the 6th grade onward, no student reasoning at level 3 and level 4 was found for the concept of sample space.

The concept of sample space is one of the basic concepts of the subject of probability. The fact that level 3 and level 4 were not observed for the concept of sample space showed that this concept was not fully understood. It was observed that the students did not consider all the situations requested in the relevant item. Furthermore, it was observed that the number of all cases was tried to be determined by writing random numbers, and that those who used the listing method wrote two situations corresponding to the same situation. To the following question in the measurement tool "There are 4 green, 3 red and 2 yellow balls in a bag. After shaking this bag, a ball is selected. What colour ball do you think will be selected? Explain why.", mostly the answer "green ball" was given. It was observed that they tended to select the one with the highest probability rather than considering all the outcomes, to focus on the possible outcome in a single experiment and to calculate quantitative probability in sample space questions. The reasons for this can be that the students may have perceived it as a similar question because they were familiar with the theoretical probability calculation questions and answered in this direction, or that they had the misconception of the result approach in the literature. The low levels of probabilistic reasoning found for the concepts of sample space, experimental probability of an event, conditional probability and independence have also been reported in the relevant literature (Bulut, 2001; Konold et al., 1993; Memnun et al., 2010; Tarr and Jones, 1997).

The learning outcome related to the concept of the experimental probability of an event (MoNE, 2009) is addressed at grade 8. When the students' levels were examined according to their grade levels, it was seen that most of the level 4 thinkers were 8th grade students. However, when the 8th grade students were examined within themselves, it was seen that more than half of them were level 1 thinkers. In the question item related to the concept of experimental probability of an event, it was observed that the students considered the given situation theoretically, without distinguishing whether it was experimental or theoretical. The fact that they were unaware of the difference between experimental and theoretical probability and that they used theoretical probability calculations instead of experimental probability calculations is also supported by Çakmak and Durmuş (2015). Some sample student answers to the 4th question in the probabilistic reasoning test are as follows: "If he missed only 1 out of 10 shots, this football player is a good football player", "I think he will score", "It depends on the angle and speed of hitting the ball". When these responses were examined, it was revealed that their answers were influenced by their daily life and school experiences, intuitions and beliefs. One of the reasons that make it difficult to learn the subject of probability is students' making such wrong connections. There are many studies on this subject in the literature (Koirala, 2003; Sezgin Memnun, 2008; Sharma, 2005, 2012; Williams and Amir, 1995).

The concept of theoretical probability of an event was found to be one of the concepts for which the students exhibited a high level of probabilistic reasoning. Since the related learning outcome (MoNE, 2009) is started to be addressed from 6th grade onwards, it was expected that level 4 thinkers would be in the majority, but level 3 thinkers were more frequently encountered. Students generally preferred to answer without determining quantitative probability. The reason for their not reaching level 4 is that they did not use quantitative reasoning. It can be claimed that they were insufficient in demonstrating quantitative reasoning due to misconceptions about the concepts of ratio, fraction and set (Çakmak and Durmuş, 2015; Çelik and Güneş, 2007; Gürbüz, 2006; Memnun et al., 2010; Sezgin Memnun, 2008; Sharma, 2012).

The concept of probability comparisons was found to be another concept for which the students exhibited a high level of probabilistic reasoning. Similarly, in this concept, which is closely related to the theoretical probability of an event, level 3 thinkers were more frequently encountered. The reason for their not reaching level 4 could be their misconceptions about the concepts of ratio, fraction and set and in the subject of making fractional comparisons (Çakmak and Durmuş, 2015; Çelik and Güneş, 2007; Gürbüz, 2006; Memnun et al., 2010; Sezgin Memnun, 2008; Sharma, 2012;).

When the levels of reasoning exhibited for the concept of conditional probability were examined, it was observed that the students were mostly at level 1. Although in the relevant question in the measurement tool “Elections for president and vice president will be held in your class and there are five candidates. Candidates: Ayça, Murat, Seda, Nedim and you. All the five candidates are considered to have an equal probability of winning. Suppose that you have determined the president. What can be said about the probability of the vice president’s being a boy or a girl? Why? Explain.”, it was clearly stated that the election of the vice president would be held, most of the students were observed to attempt to solve the problem for the president. Thus, they neglected the size of the sample space. When the answers given to the question in the measurement item “Since it is known in the experiment of tossing two coins that they both look the same, what can be said about the probability of one being a tail and the other being a tail? Explain.” were examined, it was seen that the tossing of two separate coins was considered as events that did not affect each other, like the tossing of a single coin. It was revealed that there was a misconception of equal probability bias in students who examined it as an independent event. It was observed that students might have misconceptions due to the effect of sample size in their answers and that sufficient reasoning was not performed. When the studies on the concept of conditional probability are examined, it is seen that this concept has been handled separately.

The objectives related to the concept of independence (MoNE, 2009) are addressed at grade 8. However, it was revealed that more than half of the 8th grade students remained at level 1. When the answers of the students were examined, it was seen that the sequential events were related, and they often contradicted their intuitions and beliefs. To the question in the measurement tool “A coin is tossed five times and the result is HHHHH. Are heads or tails more likely on the next toss? Please explain. (H: Heads, T: Tails)”, students gave answers without resorting to quantitative reasoning and just by evaluating past trials and considering their representativeness status such as “It was always heads, so tails will come this time” or “It was always heads, so heads will come again”. Similarly, they gave answers to the question “For families with five children, which order of birth is BGGBG or BBBBB more common? Please explain. (B: Boy, G: Girl)” such as “Five boys consecutively; it is not possible” or “It is more common to be in a mixed order, like a boy, a girl” according to their representativeness status, and they were affected by negative sequentiality. With their answers such as “It was always heads, so heads will come again” and “It started with a boy and continues with a boy, it is so in my relatives”, some students were under the effect of positive sequentiality, although their number is not as high as the ones under the effect of negative sequentiality (Çelik and Güneş, 2007; Fast, 1997; Fischbein and Scnarch, 1997; Gürbüz et al., 2014; Kazak, 2010b; Konold et al., 1993; Özdemir, 2017; Rubel, 2007; Sharma, 2005; Tarr and Jones, 1997; Williams and Amir, 1995)

When the probabilistic reasoning levels of the students were examined for the concepts, it was found that they had different levels of probabilistic reasoning across the concepts. The same student was found to be at level 1 for the concepts of sample space, experimental probability of an event, conditional probability and independence, but at level 3 for the concepts of theoretical probability of an event and probability comparisons. This could stem from their familiarity with the theoretical probability calculation questions addressed in the learning outcomes in the curriculum. It was observed that the students’ misconceptions such as the result approach, representation shortcut, negative sequentiality effect, positive sequentiality effect, equal probability bias, were not effective in their levels of probabilistic reasoning for the concepts of theoretical probability of an event and probability comparisons. Students’ reasoning in the sample space, experimental probability of an event, dependent probability and independence remain at low levels due to the students' readiness level, their misconceptions, the age factor, and the inadequacy of their reasoning skills (Çakmak and Durmuş, 2015; Fast, 1997; İlgün, 2013; Sezgin Memnun, 2008).

Since gender is an important factor in determining the mathematics performance (Halat, 2006), probabilistic reasoning levels were examined according to the gender variable in the current study. Thus, whether there was a significant correlation between the level of probabilistic reasoning and the gender variable was investigated. As a result of the analysis, no significant correlation was found between the reasoning levels of the students and gender for the other concepts, except for the concept of sample space. A weak correlation was found between sample space and gender. This result is supported by the related studies in the literature (Bulut et al., 2002).

It has been clearly proven by the past research that the age factor affects the teaching of the subject of probability. Therefore, in the current study, the correlation between probabilistic reasoning levels and grade level was investigated. As a result of the analysis, a significant correlation was found between probabilistic reasoning levels and grade level for all the sub-concepts. When the studies of Fischbein and Scnarch (1997), Sezgin Memnun (2008), Kazak (2010b), Gürbüz et al. (2014) are examined, it is seen that misconceptions about the subject of probability decreases as the age increases. Thus, the finding of the current study concurs with the literature.

In the current study, it was also investigated whether there was a correlation between probabilistic reasoning levels and mathematics achievement. As a result of the analysis, a significant correlation was found between probabilistic reasoning levels and mathematics achievement for the concepts, except for the concepts of experimental probability of an event and independence. This result is parallel to the results reported in the study by Gürbüz and Erdem (2014).

Implications

In the current study, it was investigated what the levels of the 6th, 7th and 8th grade middle school students' probabilistic reasoning were and whether these students' reasoning levels were related to the variables of gender, grade level and mathematics achievement. As a result of the study, probabilistic reasoning levels of the students participating in the study were determined for the six sub-concepts. In general, the students were found to have low levels of probabilistic reasoning. With this study, it has been observed that the problems in teaching the subject of probability still continue. In light of the findings of the current study, the following suggestions can be made to overcome the difficulties experienced in the subject of probability:

1. This study, which investigated probabilistic reasoning levels, can be replicated at all middle school grade levels according to the updated curriculum.
2. This study can be improved in such a way as to explore via interviews all the sub-concepts over two questions, one conditional probability question and one independence question.
3. This study can be replicated with primary school students in order to investigate the effect of the curriculum on probabilistic reasoning.
4. Teaching should be supported with concrete situations in order to prevent students from holding common prejudices and beliefs regarding experimental probability of an event and independence.

Genişletilmiş Özet

Giriş

NCTM (2000), olasılık öğretiminin erken yaşlarda başlaması gerektiğini vurgulamış ve okul öncesi yaş grubundaki çocukların olasılık kavramlarıyla informal olarak karşılaştıklarını ve günlük hayatta kullanılan ifadelerle olasılığı karşılamaya başladığını ifade etmiştir. Matematiği anlayabilmek için akıl yürütme becerisinin gerekli olduğu gibi matematiğin dalı olan olasılığı da anlayabilmek için akıl yürütme becerisi temel gerekliliktir. Olasılıksal akıl yürütme, olasılıksal süreçleri anlayabilmek ve açıklamaktır. Olasılıksal akıl yürütme, rastlantısal olaylara benzer model yapabilmeyi, olasılıkları tahmin etmek için uygun veriyi belirleyebilmeyi, bir problemi çözerken ilişkili durumları kullanabilmeyi ve öznel olasılıkların hangi durumlarda kullanılabileceği konusunda düşünmeyi içermektedir (Jolliffe, 2005).

Yöntem

Araştırmada ortaokul öğrencilerinin olasılıksal akıl yürütme beceri düzeylerini belirlemek için betimsel tarama modeli; öğrencilerin olasılıksal akıl yürütme becerileri ile cinsiyet, sınıf seviyesi ve matematik başarısı değişkenleri arasında ilişki olup olmadığının araştırılması için ilişkisel tarama modeli kullanılmıştır. Araştırmaya 6, 7 ve 8. sınıf düzeyindeki 286 öğrenci katılmıştır. Araştırmanın yapıldığı yıl olan 2014 yılında 2013 matematik öğretim programı uygulanmaktaydı. Müfredatlar arasında kademeli geçiş olduğu bir zaman diliminde veriler toplandığı için 2009 müfredatına göre eğitim gören tüm sınıflar araştırmaya dahil edilmiştir. Araştırmaya bu yüzden 5. sınıflar dahil edilmemişlerdir.

Uygulanan 2009 yılı öğretim programında olasılık alanında yer alan konular sınıf düzeylerine göre 6. sınıf düzeyinde olay çeşitleri; 7. sınıf düzeyinde ayrık ve ayrık olmayan olay, permütasyon; 8. sınıf düzeyinde bağımlı ve bağımsız olasılık, kombinasyon olarak yer almaktadır. Değişen öğretim programı olan 2013 yılı programında 6. ve 7. sınıf düzeylerinde olasılık alanına ait kazanımlar kaldırılarak sadece 8.sınıf düzeyinde olasılık alanına yer verilmiştir.

Öğrencilerin olasılıksal akıl yürütme düzeylerini belirlemek için süreç içerisinde Jones, Thornton, Langrall ve Tarr (1999) tarafından geliştirilen rubrik ve teorik yapısı temel alınarak yeni bir ölçme aracı oluşturulmuştur. Jones ve diğ. (1999) olasılıksal akıl yürütme becerilerini; örnek uzay, bir olayın deneysel olasılığı, bir olayın teorik olasılığı, olasılık karşılaştırmaları, bağımlı olasılık ve bağımsızlık kavramları altında, hiyerarşik olarak ilerleyen 4 düzey tanımlamaktadırlar. Ölçme aracında yer alan her bir alt kavramda 4 düzey belirlendiği için maddelerin puanlaması 1 ile 4 arasında yapılmıştır. Her bir madde için en düşük puan 1, en yüksek puan ise 4 olarak kodlanmıştır. Ölçme aracındaki soru ve öğrencilerin örnek yanıtlarıyla incelendiği detaylı bir rubrik hazırlanmıştır.

Sonuç

Araştırmanın bulgularına göre öğrencilerin çoğunluğunun örnek uzay, deneysel olasılık, bağımlı olasılık ve bağımsızlık alt kavramları için 1. düzey akıl yürütme becerisine sahip oldukları belirlenmiştir. Bir olayın teorik olasılığı ve olasılık karşılaştırmaları alt kavramlarında ise öğrencilerin çoğunluğunun 3. düzey akıl yürütme becerisine sahip oldukları belirlenmiştir.

Örnek uzayda akıl yürütme ile cinsiyet arasında anlamlı bir ilişki olduğu bulunmuştur. Olasılıksal akıl yürütmenin diğer alt kavramları ile cinsiyet arasında istatistiksel olarak bir ilişki bulunamamıştır.

Olasılıksal akıl yürütmenin alt kavramları olan örnek uzay, bir olayın deneysel olasılığı, bir olayın teorik olasılığı, olasılık karşılaştırmaları, bağımlı olasılık, bağımsızlık ile sınıf düzeyi arasında da anlamlı bir ilişkinin olduğu bulunmuştur. Bu ilişki, örnek uzay, olasılık karşılaştırmaları, bir olayın teorik olasılığı, bağımlı olasılık alt kavramları için zayıf; bir olayın deneysel olasılığı, bağımsızlık için orta düzey olarak belirlenmiştir.

Matematik başarısı ile olasılıksal akıl yürütmenin alt kavramlarından örnek uzay, bir olayın teorik olasılığı, olasılık karşılaştırmaları ve bağımlı olasılık arasında anlamlı bir ilişki bulunmuştur. Matematik başarısı ile olasılıksal akıl yürütmenin diğer alt kavramları olan bir olayın deneysel olasılığı ve bağımsızlık arasında anlamlı bir ilişki bulunamamıştır. Matematik başarısı ile bağımlı olasılık ve olasılık karşılaştırmalarında orta; bir olayın teorik olasılığı ve örnek uzay ile zayıf bir ilişki belirlenmiştir.

Tartışma ve Öneri

Araştırmadan elde edilen sonuçlara göre, bir olayın teorik olma olasılığı ve olasılık karşılaştırmaları akıl yürütme düzeyleri 3. düzeyde yoğunlaşmıştır. Örnek uzay, bir olayın deneysel olasılığı, bağımlı olasılık ve bağımsızlık alt kavramlarındaki akıl yürütme düzeylerinin düşük düzeylerde yoğunlaşması literatürdeki ilgili çalışmalarla paralellik göstermiştir (Bulut, 2001; Çelik ve Güneş, 2007; Konold ve diğ, 1993; Memnun ve diğ, 2010; Tarr ve Jones 1997).

Cinsiyet faktörü matematik performansını belirlemede önemli bir faktör olduğundan (Halat, 2006), bu çalışmada olasılıksal akıl yürütme beceri düzeyleri cinsiyet değişkenine göre incelenmiştir. Analiz sonucunda, örnek uzay alt kavramı dışındaki diğer alt kavramlarda öğrencilerin akıl yürütme beceri düzeyleri ile cinsiyet arasında bir ilişki bulunamamıştır. Örnek uzay ve cinsiyet arasında zayıf bir ilişki bulunmuştur. Bu durum literatürdeki ilgili çalışmalarla paralellik göstermektedir (Bulut ve diğ, 2002). Matematik başarısı ile bir olayın deneysel olasılığı ile bağımsızlık alt kavramları arasında ilişki olmaması beklenen sonuçlardan biridir. Çünkü öğrencilerin bu kavramlarda öğrenme güçlüğü yaşadıkları gözlemlenmiştir (Çelik ve Güneş, 2007; Konold ve diğ, 1993; Memnun ve diğ, 2010; Sezgin Memnun, 2008; Tarr ve Jones, 1997). Bu araştırma nicel bir çalışmadır. Daha detaylı bulgular ortaya koymak için nitel çalışmalar yapılabilir.

Araştırmanın Etik Taahhüt Metni

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